Modeling soil-fluid and fluid-soil transitions with applications to tailings

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ABSTRACT: While geotechnical engineers concentrate on soil with a grain skeleton which is subject to effective stresses, hydraulic engineers concentrate on fluids and fluidized mixtures. Similarly, researchers in geomechanics may focus on soft soils, leaving fluidized materials to researchers in computational fluid dynamics. Hence, for a long time the transition from a soil skeleton to a fluid with soil particles, and vice versa, remained unsolved. Recently, a breakthrough was created by modeling this transition. The theory is summarized in this paper, even if the focus is on numerical applications. Before addressing the modeling of transition, the employed numerical method, the so-called material point method, is applied to sand capping problems and a collapsing water column for the purpose of validation. Afterwards, the collapse of an underwater slope is treated. Here, breaching is considered which occurs in case of steep slopes in dense dilative sand. Whereas classical finite element analyses tend to stop at the onset of a failure, the analysis with the material point method gives also results for post-failure flow and re-sedimentation.

1 INTRODUCTION

The deposition of tailings in ponds extending across several square kilometers with depths of up to 50 meters represents a demanding engineering problem that requires efficient and safe solutions. Typical tailings generally feature low strengths making it difficult to seal off tailings through sand capping. Once the sand cap is in place, it exhibits a very slow consolidation process and thus a very slow increase of strength with time. Clearly, such a material, tailings-related problems such as the installation of a sand cap respectively, require consideration of a two-phase material of soil and water including the transition from water-saturated soil to soil grains suspended in water.

The material point method (MPM) has been applied in recent years to large-deformation problems of granular flow and soil mechanics such as silo discharge, anchor pull-out, slope failure, pile jacking and driving (Więckowski et al. 1999, Więckowski 2004, Coetzee et al. 2005, Beuth et al. 2011, Beuth 2012, Al-Kafaji 2013). Recently, the consideration of large deformation processes involving mixtures of soil and fluid has been incorporated (Więckowski 2013). The latter allows considering problems such as consolidation of saturated soil, flow of water through porous media as well as erosion and sedimentation processes.

In this paper, numerical analyses with MPM are presented that involve soil and fluid as well as mixtures of both materials to demonstrate the applicability of the method to tailings-related problems.

In the following, firstly, numerical analyses involving undrained soft soil behavior are presented. Validation of the method is provided through comparison of results with an analytical
solution for the problem of placing a sand ridge onto tailings. Furthermore, placing of a heap of sand onto tailings is treated. Section 3 deals with the modeling of fluids with MPM. Validity of the fluid formulation is demonstrated by means of the problem of a collapsing water column. The extension of MPM to soil-fluid mixtures is treated in Section 4. Its application is demonstrated by means of the problem of breaching.

2 MATERIAL POINT METHOD FOR UNDRAINED SOFT SOILS

MPM can be considered as an extension of the classical updated Lagrangian finite element method (UL-FEM). For the UL-FEM, a body is discretized by finite elements that follow the deformations of the body. In case of large deformations of a body, the finite element mesh might eventually experience severe distortions, which lead to numerical inaccuracies and can even render the calculation impossible.

In MPM, a body is discretized by a cloud of material points that moves through a fixed finite element grid. Thereby, the material points capture the arbitrary large deformations of the body without the occurrence of severe deformations of finite elements. Material and state parameters of the body as well as applied loads are stored and transported in material points whereas the mesh does not store any permanent information.

For MPM, the underlying finite element mesh is used (as with UL-FEM) to solve the system of equations of motion. Once displacements, stresses and strains are computed at locations of material points, the mesh is usually reset into its original state. However, it might also be changed arbitrarily.

For further information on MPM the reader is referred to (Sulsky 1994, Więckowski 1999, Więckowski 2004, Al-Kafaji 2013). A detailed description of the method would exceed the scope of this paper.

In this section on undrained behavior of tailings, water-saturated soil will be considered as a single-phase material. This allows for a validation of the code on the basis of boundary value problems with an analytical solution, as will be done in Section 2.1.

As usual in soil mechanics, undrained material behavior is modeled on the basis of a so-called total stress analysis, in which shear stresses are limited by the introduction of undrained shear strength, being generally denoted as \( s_u \). On performing numerical analyses, the undrained shear strength is embedded in an elastic-plastic material model. Both, the so-called Von Mises yield criterion and the resembling Tresca yield condition can be applied. In this study the latter is used.

2.1 Validation for sloping sand cap on tailings

The creation of a sand cap on top of tailings will inevitably involve a spatial variation of the thickness of this cap. No doubt, tailings will be able to carry such a cap as long as these spatial variations are small, but the bearing capacity of soft tailings will not allow for strong variations. Hence, analyses will have to be performed to assess the bearing capacity of tailings with respect to thickness variations of the sand cap. Considering soft tailings, pre-failure squeezing will be considerable, causing significant geometry changes of the sand-tailing interface even well before collapse. No doubt, the analysis of such large-deformation problems requires an advanced numerical analysis. On the other hand, small-deformation based analytical solutions remain important for the validation of numerical approaches such as MPM. In this section, this method will be validated by focusing on the bearing capacity of soft tailings under a ridge of sand, as illustrated in Figure 1a.

For soft tailings, the sand-ridge problem is of special interest as Davis & Booker (1973) published an analytical solution for the bearing capacity of soil layers in which the undrained shear strength increases linearly with depth. As their solution is restricted to small-deformation problems without geometry changes, this will also be assumed in the present numerical MPM analysis. Indeed, once this method has been validated for quasi-static small-deformation analyses, it can be applied with some confidence to dynamic large-deformation problems.

Material parameters roughly reproducing those of typical soft tailings are listed in Table 1. At the top of the tailings the undrained shear strength is taken to be 100 [N/m²]. A significant in-
crease with depth is considered by adding 500 [N/m²] for each meter of depth. The shear modulus also increases with depth and is assumed to be a hundred times larger than the undrained shear strength. At the surface this implies a shear modulus of 10 [kN/m²] and its increase with depth is 50 [kN/m²] per meter of depth. For undrained conditions, tailings cannot be compressed and this is modeled by taking the undrained Poisson ratio nearly equal to 0.5. For numerical reasons, a nearly incompressible material is modeled by adopting a value of 0.495.

Table 1. Soft tailings material parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Undrained shear strength at top</td>
<td>$s_{u,\text{top}}$ 100 [N/m²]</td>
</tr>
<tr>
<td>Increment of undrained shear strength with depth</td>
<td>$s_{u,\text{increment}}$ 500 [N/m²/m]</td>
</tr>
<tr>
<td>Shear modulus at top</td>
<td>$G_{\text{top}}$ 10 [kN/m²]</td>
</tr>
<tr>
<td>Increment of shear modulus with depth</td>
<td>$G_{\text{increment}}$ 50 [kN/m²/m]</td>
</tr>
<tr>
<td>Undrained Poisson ratio</td>
<td>$\nu_u$ 0.495</td>
</tr>
</tbody>
</table>

Placement of a long sand ridge with a total cross-sectional width of 2 [m] on top of tailings is considered, which yields a plane-strain symmetrical problem. A symmetrical half is shown in Figure 1a. As a full 3D code is used in this study, a slice of tailings of 0.2 [m] width is shown. This slice is divided into 4-noded tetrahedral finite elements. The thin sand fill is modelled by the application of a distributed load at the surface of the finite element mesh, as also shown in Figure 1a. Considering a tailings pond with a water level well above the tailings the submerged weight of the sand cap is applied. During the calculation, the slope of the sand cap is stepwise increased. In Figure 1b, the applied slope is plotted against resulting settlements at the center of the sand cap. For a slope inclination of 1:8, a ‘run-away’ failure is computed.

Davis & Booker (1973) present an analytical solution for the bearing capacity of clay taking into account an increase of strength with depth. For the considered sand ridge, they predict a maximum sand height at the foot of the slope of 5.7 [cm] and at the head of the slope of 16.7 [cm], corresponding to an inclination of 1:9. Ultimate sand heights of 6.3 and 18.2 [cm] and a slope inclination of 1:8 result from the numerical analysis. The good agreement with the analytical solution gives confidence in the used MPM.

Figure 2 indicates that a rather shallow failure mechanism evolves. At failure, only the upper 50 [cm] of the tailings material experience deformations.

Figure 1. (a) Geometry and mesh of the discretized sand ridge and applied distributed load; (b) slope of the ridge plotted against the settlements at the center of the sand cap.

2.2 Preliminary study of axisymmetric sand heap on tailings

In addition to the installation of a sand ridge, the case of a sand heap with a radius of 1 [m] on tailings is considered in the following. The sand cap is modeled as in Section 2.1 by a distributed load. The same material parameters were used for the tailings material. Large deformation ef-
fects are not considered. The placement of a sand heap is treated as an axisymmetric problem. Only a slice of 20 ° with a radius of 2 [m] and a height of 1 [m] is discretized. The geometry and discretization of the problem are shown in Figure 3a. The same finite element discretization as in the analysis described above is used.

![Figure 2. Relative magnitude of velocities at failure for sand ridge.](image)

In Figure 3b, the applied slope is plotted against resulting settlements at the center of the sand cap. To the authors' knowledge, no analytical solution exists to this problem. The analysis renders a maximum thickness of the sand layer at the foot of the slope of 6.2 [cm] and a maximum thickness at the head of the slope of 21.7 [cm] which corresponds to an inclination of 1:7. As expected, a steeper slope can be realized compared to the plane strain problem, as the load is distributed with increasing distance from the center across a larger area. Obviously, as can be seen in Figure 4, failure of the tailings material involves progressive mobilization of its shear strength across a wider failure surface compared to the sand ridge. Likewise, in the numerical analysis a rather shallow failure mechanism is predicted reaching at failure a depth of only approximately 50 [cm], as indicated in Figure 4.

![Figure 3. (a) Geometry and mesh of the discretized sand heap and applied distributed load; (b) slope of the heap plotted against the settlements at the center of the sand cap.](image)
3 MATERIAL POINT METHOD FOR FLUIDS

While solid mechanics problems are generally solved with Lagrangian methods such as ULFEM, Eulerian methods such as the finite volume method are applied to problems of fluid flow. With the latter, basically, fluid velocities are computed at fixed points in space, for example the nodes of a stationary mesh, rather than at material points of the fluid.

MPM which can also be considered as a mixed Lagrangian-Eulerian approach is well suited for both types of problems. In fact, fluid simulations with MPM go back to its very beginnings in the 1950's when it was developed by Harlow under the name of particle-in-cell method (Harlow 1964). Since then it has been applied successfully to various problems of fluid flow and fluid-structure-interaction (Gwilkey 2007).

As in the analyses of Section 2, a fluid is discretized by a cloud of material points that moves through a background finite element mesh. The equations of motion are assembled and solved at the nodes of the mesh, stresses and strains are computed at the location of material points. Contrary to Section 2, the equations of motion are now based on the Navier-Stokes equations which describe the motion of fluids.

Obviously, modeling fluids forms a prerequisite for considering soil-fluid mixtures. In the following, the application of MPM to the problem of a collapsing water column is presented for validation purposes.

Water is modeled as a viscous liquid. As with the previous analyses, a slight compressibility is considered for numerical reasons.

3.1 Validation for collapsing water column

A column of water is released from a box by suddenly removing one of its sides. The surfaces of the box are assumed to be smooth. The water spreads out onto a flat smooth surface. The problem has been investigated experimentally by Martin & Moyce (1952) and numerically by Zienkiewicz et al. (2005).

The geometry of the problem is shown in Figure 5. The water column initially has a height $H_0$ of 7 [m] and a width $W$ of 3.5 [m]. A dynamic viscosity $\mu$ of $0.8905 \times 10^{-6}$ [kN/m²·s], a bulk modulus $K$ of $2.1284 \times 10^6$ [kN/m²] and a mass density $\rho_i$ of 1,000 [kg/m³] are used.

The position of a water particle located initially at the top right corner of the column has been traced. In Figure 6, the relative column height computed from the particle's vertical position and the initial column height $H_0$ is plotted versus time. Clearly, good agreement is obtained with the experimental results of Martin & Moyce (1952).

This problem can also be solved by the finite element method formulated in the purely Lagrangian format even though severe mesh distortions occur. The purely Lagrangian solution of
the considered problem has been shown by Zienkiewicz et al. (2005). Comparison of the results in Figure 7 shows good agreement between the two numerical analyses.

Figure 5. Geometry and mesh of the water column in its initial state and extend of discretized space.

Figure 6. Relative column height plotted versus time.

Figure 7. At top, collapsing water column computed with MPM. At bottom, UL-FEM computation (Zienkiewicz 2005). Results at $t = 0.65$ [s] and 1.60 [s] are shown.

4 MATERIAL POINT METHOD FOR TWO-PHASE TRANSITION PROBLEMS

This novel extension of the MPM has been recently developed by one of the co-authors, Z. Więckowski, in the frame of the EU-FP7 Geo Fluid project. The details of the problem formulation...
tion and numerical model are shown in (Więckowski 2013).

In order to analyze state transition problems for a porous saturated body like fluidization and sedimentation, a two-phase formulation with two sets of material points representing solid and fluid is introduced into MPM. Now, instead of one set of equations of motion as in Sections 2 and 3, equations of motion for both, solid and fluid phases are solved, rendering separate velocity fields for solid and fluid. The equations of motion for both the media can be stated as follows:

\[
(1 - n) \rho_s \frac{\partial v_i}{\partial t} = \sigma_{ij}^* - (1 - n) p_j \rho_s + \mu_{st} + (1 - n) \rho_s g_i
\]

\[
n \rho_f \frac{\partial w_i}{\partial t} = -n p_j + n s_{ij} - f_{di} + n \rho_f g_i
\]

where \(v_i\) and \(w_i\) are the velocity vectors for the solid and fluid, respectively. \(n\) denotes the porosity of the soil defined as the ratio of void volume with respect to reference volume. \(\rho_s\) and \(\rho_f\) denote the mass density of the soil skeleton and of the fluid, respectively. \(\sigma_{ij}^*\) is the effective stress tensor, \(p\) is the pore pressure of the fluid, \(s_{ij}\) the deviatoric part of the stress tensor, \(g_i\) the vector of acceleration of gravity. \(f_{di}\) denotes the drag force exerted by the fluid on the unit volume of the solid. The tortuosity of the fluid flow path is not taken into account in Equations 1 and 2.

Stresses are computed from the following constitutive relations for the soil:

\[
\sigma_{ij}^* = \left( \sigma_{ij} - \rho_s \frac{\partial \varepsilon_{ij}^{vol}}{\partial t} \right)
\]

(3)

and for the fluid:

\[
\frac{\partial \rho}{\partial t} = -K_f \frac{\partial \varepsilon_{ijkl}^{vol}}{\partial t}
\]

(4)

\[
s_{ij} = 2 \mu_v \frac{\partial \varepsilon_{ij}^{vol}}{\partial t}
\]

(5)

where \(\sigma_{ij}^*\) is the objective rate of stress tensor, \(\varepsilon_{ij}\) is the rate of deformation tensor and \(\varepsilon_{ijkl}^{vol}\) denotes the volumetric strain for the fluid phase. The viscosity is considered to depend on the volume fraction of solid particles suspended in the fluid according to, for example (Beenakker 1984):

\[
\mu_v = \mu \left(1 + 5/2 \phi + 5.2 \phi^2\right)
\]

(6)

where \(\phi\) denotes the volume fraction of the particles suspended in the fluid. This formula is derived for a fluid with spherical particles suspended in it.

The interaction between solid and fluid phase is described by means of Ergun’s law (Ergun 1952):

\[
f_{di} = \frac{n^2 \left( \mu / K (w_i - v_i) + n \rho_f F / \kappa^{1/2} \right) |w - v| (w_i - v_i)}
\]

(7)

Here, \(\kappa\) is the permeability of the soil and \(F\) Forchheimer’s coefficient. These coefficients are defined as:

\[
\kappa = \frac{n^3 d^2}{a(1 - n)^2}, \quad F = \frac{b}{\sqrt{a n^{3/2}}}
\]

(8)

where \(d\) denotes the average particle size of the solid, \(a = 150\), and \(b = 1.75\).

The material can take on one of two states which are identified by means of the material’s porosity. In the low porosity state, the grains of the soil skeleton are in contact and liquid flows
through the pores, the effective stresses in the soil are non-zero. In the high porosity state, the solid material is fluidized and there is no direct contact between its (suspended) grains inside the liquid, the effective stresses are zero. The cases of dry soil and pure fluid represent limits of these two states. The case of partially saturated soil can be considered as a case of low porosity.

Consideration of the solid phase: In the process of the soil-fluid mixture changing from a saturated to a liquefied state, the effective mean stress of the soil skeleton tends towards zero, while the porosity of the soil is increasing. The moment when the effective mean stress is zero, the soil can be considered as liquefied. In the reverse process of sedimentation, the porosity of the soil is decreasing. However, it cannot be controlled by tracing the effective mean stress. Therefore, a threshold porosity, \( n_0 \), is introduced at which the onset of contact between solid grains and thus non-zero effective stresses are assumed. The change of porosity is computed using the distribution of solid material points.

Consideration of the fluid phase: The fluid volume fraction is traced in order to detect the creation of voids inside the fluid due to the deformation process. When the fluid (locally) becomes a cloud of separated drops, the pressure and other stress components of the fluid vanish. Once the amount of voids diminishes, the mass concentration of the fluid reaches a threshold value and the non-zero stresses of the fluid are computed by means of Equations 4 and 5.

4.1 Example of state transition analysis

State transition phenomena like fluidization and the onset of sedimentation are analyzed considering the problem of a collapsing sand column submerged in a water reservoir. The example is described in detail in (Więckowski 2013). A cross-section through the reservoir and the initial configuration of the sand column are shown in Figure 8. Considering the symmetry of the problem, only half the region is discretized. The reservoir is discretized by a regular structured 2D mesh of triangular elements.

It is assumed that the sand is fully saturated. It is modeled by means of an elastic-viscoplastic material model using the Drucker-Prager yield condition (Więckowski 2004). The material parameters of the sand and water are listed in Tables 2 and 3. The bulk modulus of the fluid is chosen 100 times smaller than for water in order to increase the speed of the computation.

Figure 9 shows the soil mass concentration at the stated real times of the collapse process. Only the position of material points representing the solid phase is shown.

The fluidization phenomenon and subsequent flow of the liquefied sand along the column surface can be clearly seen. This kind of flow which takes place along vertical or steep submerged slopes is known as breaching and is purposely induced in some engineering processes such as dredging of a river or sea bed. In Figure 10, later stages of the flow process are shown. After 50 [s], when the column is almost “dissolved” in water, the onset of the sedimentation process can be noticed.

![Figure 8. Initial configuration of submerged soil column. Dimensions are in [m].](image)
<table>
<thead>
<tr>
<th>Table 2. Sand material parameters.</th>
<th>Table 3. Water material parameters.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mass density</strong> $\rho_s$</td>
<td>$\rho_w$ 1,000 [kg/m$^3$]</td>
</tr>
<tr>
<td>Initial mass concentration $\rho_{ho}$</td>
<td>$\mu$ 8.905 $\times$ 10$^{-7}$ [kN/m$^2$ s]</td>
</tr>
<tr>
<td>Young’s modulus $E$</td>
<td>Bulk modulus $K$ 2.128 $\times$ 10$^4$ [kN/m$^2$]</td>
</tr>
<tr>
<td>Poisson ratio $\nu$</td>
<td>Viscosity $\gamma$ 10$^{-5}$ [1/s]</td>
</tr>
<tr>
<td>Internal friction angle $\varphi$</td>
<td>Index of power law $N$ 1</td>
</tr>
<tr>
<td>Viscosity $\gamma$</td>
<td>Grain diameter $d_o$ 0.2 [mm]</td>
</tr>
<tr>
<td>Index of power law $N$</td>
<td>Threshold porosity $n_0$ 0.5</td>
</tr>
</tbody>
</table>

5 CONCLUSIONS

In this paper, the feasibility of advanced numerical analyses on the cutting edge between geo-mechanics and hydromechanics has been demonstrated. Considering sediments, an important novelty is the modeling of state transitions in assemblies of solid particles, e.g. the erosion-type transition of a soil into a sediment suspension or inversely the sedimentation-type transition followed by consolidation. Near-surface consolidation is also of utmost importance for sand capping. Indeed, in this paper the immediate undrained response of loading tailings has been considered, but as yet we did not address resulting excess pore pressures. A proper evaluation of these pressures is needed for assessing the increasing shear strength of tailings during consolidation. On using relatively simple elastic-plastic soil models, excess pore pressures in soft tailings tend to be underestimated by a factor 0.5 and consequently more advanced models will have to be used (Beuth 2012). Subsequently, such constitutive models will have to be applied in consolidation analyses. Meanwhile, we began to do such analyses in the framework of dissertation studies. First results of such studies were published by Al-Kafaji (2013).

The treatment of mine waste in the form of tailings poses a demanding engineering problem. It requires knowledge and experience in both, geo- and hydromechanics. MPM is therefore proposed here as a powerful numerical tool to bridge the gap between the two disciplines, opening new paths to innovative solutions to this field of engineering.

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Figure 9. Soil mass concentration for some deformation phases of collapsing column.

Figure 10. Soil mass concentration for some later deformation phases of collapsing column.

7 REFERENCES


