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The Material Point Method in Slope Stability Analysis

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The Material Point Method in Slope Stability Analysis

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Abstract

The material point method (MPM) combines an Eulerian and a Lagrangian description of the dynamic behaviour of materials. In recent years it has been extended to solve problems in soil mechanics. In particular, a code has been developed to analyse the response of saturated soils (Jassim, et al., 2012). The method is well adapted to deal with large displacements. MPM has a good potential to examine the conditions leading to slope failure but, also, it is capable of following in time the evolution of the unstable mass determining its final run-out, which is a key variable to evaluate the consequences of instability. Recently, a novel feature has been implemented in the code to simulate the excavation of soil. Within this work, the Selborne experiment has been modelled in order to contribute in the validation of the new excavation feature and the Mohr-Coulomb Strain Softening constitutive model. Furthermore, the influence of the initial stresses on the generated slip surface and the post-failure behaviour is studied.

Key words: SLOPE STABILITY ANALYSIS, NUMERICAL METHODS, MPM, MODEL, LARGE DEFORMATIONS, PREDICTION, INITIAL STRESSES, EXCAVATION, STRAIN SOFTENING CONSTITIVE LAW, SELBORNE EXPERIMENT
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# Table of contents

Abstract .......................................................................................................................... iii

Acknowledgments ........................................................................................................... v

Table of contents ........................................................................................................... vii

List of figures ................................................................................................................. xi

List of tables ................................................................................................................. xiv

1 Introduction ............................................................................................................... 1
   1.1 Background and Motivation .................................................................................. 1
   1.2 Scope of this work ............................................................................................... 3
   1.3 Outline and content ............................................................................................ 3

2 State of the art ........................................................................................................... 5
   2.1 Slope stability analysis ....................................................................................... 5
      2.1.1 Introduction .................................................................................................. 5
      2.1.2 Prediction methodologies ........................................................................... 5
      2.1.3 Limit Equilibrium Methods ........................................................................ 6
   2.2 The Finite Element Methods ............................................................................. 6
      2.2.1 Introduction .................................................................................................. 6
      2.2.2 Lagrangian and Eulerian formulations ....................................................... 7
      2.2.3 Combined Methods .................................................................................... 8
      2.2.4 Meshless Methods ..................................................................................... 8
   2.3 Progressive Failure ........................................................................................... 8
      2.3.1 Introduction .................................................................................................. 8

3 The Material Point Method ................................................................................... 11
   3.1 Dynamic Material Point Method ...................................................................... 11
      3.1.1 Introduction .................................................................................................. 11
      3.1.2 Governing differential equations ............................................................... 12
      3.1.3 Discretization of the governing equations ................................................. 14
      3.1.4 Solution procedure .................................................................................... 16
      3.1.5 Boundary conditions .................................................................................. 18
The MPM in Slope Stability Analysis

3.2 Dynamic generation and dissipation of pore pressures ................................................ 19
  3.2.1 Introduction ..................................................................................................... 19
  3.2.2 Modelling two-phase problems ....................................................................... 19
  3.2.3 Finite Element Model for $v-w$ formulation ................................................... 20
  3.2.4 Two phase dynamic Material Point Method ................................................... 20

3.3 Strain Softening constitutive model ............................................................................. 24
  3.3.1 Mohr-Coulomb Strain Softening law .............................................................. 24

3.4 Excavation Feature ...................................................................................................... 25

4 The Selborne Experiment .............................................................................................. 27
  4.1 Introduction ........................................................................................................... 27
  4.2 Geology of the site ............................................................................................... 28
  4.3 Geotechnical properties ....................................................................................... 28
  4.4 Pore pressure surcharge system ........................................................................... 29
  4.5 The failure ............................................................................................................ 30
  4.6 Recent Modelling of the Experiment ..................................................................... 31

5 The Selborne Slope Modelling ...................................................................................... 33
  5.1 Introduction ........................................................................................................... 33
    5.1.1 Pre-processing software ............................................................................. 33
    5.1.2 Post processing software ........................................................................... 33
  5.2 Pre-processing ....................................................................................................... 34
    5.2.1 Geometry ................................................................................................... 34
    5.2.2 Boundary conditions ............................................................................... 34
    5.2.3 Material ................................................................................................... 34
  5.3 Loading conditions ............................................................................................... 35
  5.4 Mesh ..................................................................................................................... 35
  5.5 Calculation procedure ........................................................................................... 36
    5.5.1 Particle initialization with the $K_0$ value .................................................... 36
    5.5.2 Excavation ............................................................................................... 36
    5.5.3 Reaching the quasi-static equilibrium ...................................................... 37
    5.5.4 Increasing the pore pressure in the bottom boundary ............................... 38
  5.6 Numerical parameters ......................................................................................... 38
    5.6.1 Damping ................................................................................................... 38
    5.6.2 Time step ............................................................................................... 39

6 Results ..................................................................................................................... 41
  6.1 Introduction ......................................................................................................... 41
6.2 Initial stresses ............................................................................................................... 41
6.3 Slip surface .................................................................................................................. 42
6.4 Final displacements and velocities .............................................................................. 43
6.5 Progressive failure ....................................................................................................... 44
6.6 Stress paths and time evolution of soil parameters ...................................................... 47
   6.6.1 P1, point located in the shear band ................................................................. 47
   6.6.2 P2, point located in the mobilized soil mass ................................................... 49
7 Parametric Study .................................................................................................... 53
   7.1 Influence of the initial stresses – $K_0$ values ......................................................... 53
   7.1.1 Slip surfaces comparison ................................................................................ 55
   7.1.2 Kinematic comparison of the failure ............................................................... 55
   7.2 Residual Cohesion, $c'_r$ .................................................................................... 57
   7.3 Intrinsic permeability $\kappa$ ................................................................................. 60
8 Conclusions .............................................................................................................. 61

References ..................................................................................................................... 63

Appendixes

A1 Appendix 1: Excavation Feature ............................................................................. A
List of figures

Figure 1.1. Typical strain ranges experienced in geotechnical engineering (Mair, 1993) & redrawn in (D.J.White, et al., 2001).............................................................................................................................. 1

Figure 1.2. Aznalcóllar dam failure (Zabala, 2010) pic by (López, et al., 2004)....................... 2

Figure 1.3. Cologne's new underground tunnel collapse (Wallis, 2009)................................. 2

Figure 1.4. Landslide in the Las Colinas (U.S. Geological Survey Public Affairs Office, 2001). 3

Figure 2.1. (a) Initial configuration for Eulerian FEM. (b) Deformed configuration for the Eulerian FEM. (c) Initial configuration for Lagrangian FEM and (d) Deformed configuration for Lagrangian FEM ........................................................................................................................... 7

Figure 2.2. Conceptual scheme for the strength of the argillaceous hard soils and weak rocks (Gens, 2011)........................................................................................................................................................................ 9

Figure 3.1. How the MPM works................................................................................................ 12

Figure 3.2. Initial and deformed configuration of a continuum (Oliver, et al., 2000)......... 12

Figure 3.3. Cauchy Stress Tensor (Oliver, et al., 2000)......................................................... 13

Figure 3.4. Boundary conditions in space (Oliver, et al., 2000)............................................. 14

Figure 3.5. Particle crossing element boundary. (a) before crossing, (b) after crossing, (c) shape function (Jassim, 2013).............................................................................................................................................. 17

Figure 3.6. Moving mesh procedure (a) Initial configurations, (b) Deformed configuration with stretched, moving and compressed zones. (Jassim, 2013).............................................................. 19

Figure 3.7. Compression problem. (a) Particle integration and (b) Gauss integration (Jassim, et al., 2012).................................................................................................................................................... 22

Figure 3.8. Boundary conditions for a given two-phase problem (Jassim, 2013)................... 23

Figure 3.9. Influence of \(\eta\) in the evolution of vertical stresses (Yerro, et al., 2014)............ 24

Figure 4.1. Site Location Plan (Cooper, et al., 1998)............................................................ 27

Figure 4.2. Geological section and slope profile (Cooper, et al., 1998).................................. 28

Figure 4.3. Original State of the stresses in the ground (Cooper, et al., 1998)...................... 28

Figure 4.4. Recharge layout system (Cooper, et al., 1998).................................................. 29
The MPM in Slope Stability Analysis

Figure 4.5. Variations in pore pressure with recharge (days 2, 170 and 184) (Cooper, et al., 1998) .......................................................... 30

Figure 4.6. Surveyed cross-section of the slip surface (Cooper, et al., 1998) ..................... 30

Figure 4.7. Interpolated cross-section of the slip surface (Cooper, et al., 1998) .............. 31

Figure 4.8. Displacements with time for the different inclinometers (Cooper, et al., 1998) ..... 31

Figure 5.1. Geometry of the model ................................................................................. 34

Figure 5.2. Refinement region ......................................................................................... 35

Figure 5.3. Used mesh for the calculations. 1.0m tetrahedral elements with 4 material points. 36

Figure 5.4. Slope elevation during the excavation procedure ......................................... 37

Figure 5.5. Out of balance Forces and Kinetic Energy Error. Reaching quasi-static equilibrium. Damping coefficient: 0.75 ................................................................. 38

Figure 5.6. Oscillations of the effective vertical stresses for different damping coefficients while gravity loading ........................................................................................................... 39

Figure 6.1. (a) Horizontal effective stresses before and after excavation, (b) Vertical effective stresses before and after excavation. ............................................................... 42

Figure 6.2. (a) Horizontal effective stresses and (b) Vertical effective stresses generated without the excavation procedure (Based on Castellvi (2015) analysis) ............... 42

Figure 6.3. Shear bands: on the left hand side considering the excavation of the soil; on the right the one without the excavation procedure (Based on Castellvi (2015) analysis) . 43

Figure 6.4. Superposed shear bands of the non-excavated case (in orange), the excavated case (in dark blue) and real one observed in Selborne (in pink). .............................................. 43

Figure 6.5. Results of the simulation superimposed with the real observed displacement vectors in Selborne ............................................................................................................. 44

Figure 6.6. Velocities of the simulations .......................................................................... 44

Figure 6.7. Shear strength states around slip surface (Cooper, 1996) ................................ 45

Figure 6.8. Points along the shear band object of the study ........................................... 45

Figure 6.9. Mobilized shear angle for the points along the shear band before the experiment starts. ....................................................................................................................... 46

Figure 6.10. Mobilized shear angle along the shear band while the progressive failure is developed .................................................................................................................. 46

Figure 6.11. Initial location of the 2 points of study for the excavated geometry............. 47

Figure 6.12. Time evolution of the parameters for material point P1 ............................. 48

Figure 6.13. Stress path for material point P1 .................................................................. 48
Figure 6.14. Time evolution of the mean effective stress, $p'$, and the deviatoric stress, $q$, for material point P1 ........................................................................................................................................... 49

Figure 6.15. Time evolution of the parameters for material point P2 ........................................... 50

Figure 6.16. Time evolution of the mean effective stress, $p'$, and the deviatoric stress, $q$, for material point P2 ........................................................................................................................................... 50

Figure 6.17. Stress path for material point P2 .................................................................................. 51

Figure 7.1. Three different points of study .................................................................................... 53

Figure 7.2. Effect of the excavation for the point P1 ................................................................. 54

Figure 7.3. Effect of the excavation for the point P2 ................................................................. 54

Figure 7.4. Comparison of the slip surfaces for each $K_0$ value ............................................... 55

Figure 7.5. Superposition of the slip surfaces ............................................................................. 55

Figure 7.6. Horizontal displacements field different $K_0$ values ............................................. 56

Figure 7.7. Horizontal displacements in time for a point above the slip surface ..................... 56

Figure 7.8. Horizontal velocity in time for a point above the slip surface ............................... 57

Figure 7.9. Parametric Study for the residual cohesion. (Castellví, 2015)................................. 58

Figure 7.10. Slip surfaces for the different values of residual cohesion of the weathered clay .. 58

Figure 7.11. Run-out results for each residual cohesion ............................................................... 58

Figure 7.12. Comparison for the excavation of the soil for a residual cohesion for the weathered Gault clay of 0kPa on the left and 0.5kPa on the right................................................................. 59

Figure 7.13. Time evolution of the stresses of a given material point. (Castellví, 2015)............. 60
List of tables

Table 4.1. Peak and residual shear strength parameters. (Cooper, et al., 1998) ......................... 29
Table 5.1. Soil parameters for the simulation ............................................................................. 34
Table 5.2. Shear strength parameters for the soils ...................................................................... 35
Table 6.1. Non-Dimensional Intervals meaning ......................................................................... 41
1 Introduction

1.1 Background and Motivation

The Material Point Method (MPM) is a relatively recent numerical technique to model large deformations. It combines the best advantages of both, the Eulerian and Lagrangian to tackle large deformation problems. In geomechanics problems involving large deformations and large movement of soil masses are usually encountered such as landslides, pile penetration, tunnels or tunnel collapse. In Figure 1.1 there is presented the typical strain ranges experienced in geotechnical engineering.

![Figure 1.1. Typical strain ranges experienced in geotechnical engineering (Mair, 1993) & redrawn in (D.J.White, et al., 2001)](image)

Landslide analysis is also an essential part for risk assessment and these methods are an emerging tool to predict the character of the failure and give a quantitative estimation of the post failure run-out, including the travel distance and the velocity. Some of the catastrophic consequences of the landslides are presented in the following figures.

Figure 1.2 presents the dam progressive failure that occurred in Aznalcóllar, Spain, in April of 1998. The rock fill dam slid forward and released a flow of acid-saturated tailings. (Alonso, et al., 2006).

Figure 1.3 shows the new underground line tunnel collapse in Cologne, Germany. Failure of the excavation caused complete collapse of one building last week on Tuesday 3 March and claimed the lives of two residents in the partially collapsed apartment buildings either side (Wallis, 2009).

In Figure 1.4 it is presented a massive landslide occurred in the Las Colinas neighbourhood of Santa Tecla, El Salvador, as a result of the M=7.6 earthquake of January 13, 2001. The landslide buried many houses in the neighbourhood under tons of earth.
The MPM in Slope Stability Analysis

Figure 1.2. Aznalcóllar dam failure (Zabala, 2010) pic by (López, et al., 2004)

Figure 1.3. Cologne's new underground tunnel collapse (Wallis, 2009).
1.2 Scope of this work

The principal aim of this work is to contribute to the validation of the MPM with the simulation of the Selborne cutting experiment. The Selborne cutting experiment was well instrumented and documented so the geometry after and before the failure is known, also the pore pressures evolution and how the progressive failure developed. Within this work the Selborne experiment is modelled and compared to the real results observed in the site. Additionally, the influence of the initial stresses on the slope stability and the post failure behaviour in the soil is studied.

1.3 Outline and content

A general overview of the existing literature about the recently developed work concerning the slope stability analysis is given in Chapter 1. Also a review of the most important existing research work regarding Numerical Methods used to model large deformations is presented. Finally, a brief review of the concept of progressive failure, present in brittle materials like over-consolidated clays, is given.

In Chapter 2 the Dynamic Material Point Method is presented according to (Zabala, 2010) and (Jassim, 2013). As the most common in geotechnical engineering is to find problems and situations where coupling between the solid and fluid phase, there is presented the extension of the MPM to the two-phase problem according to (Jassim, 2013). Moreover, the Strain Softening Mohr-Coulomb constitutive model used to simulate the brittle behaviour of the Selborne over-consolidated clay is presented according to (Yerro, et al., 2014).

Chapter 3 summarizes the Selborne Experiment based in all the information gathered in (Cooper, 1996), (Cooper, et al., 1998) and (Grant, 1996). The experiment consisted in induce the failure of a slope by increasing the pore pressure by means of wells. Information about the geology and the geotechnical properties of the site, the pore pressure surcharge system, and a description of the failure are provided.
In Chapter 5, the geometry, boundary conditions, material and loading conditions used to model the experiment are presented.

Chapter 6 presents the results of the simulations. Initial stresses, the slip surface generated, displacements and run-out observed are presented. Moreover, a complete analysis regarding the progressive failure is presented along with the stress paths for the material points along the shear band.

Besides, a parametric study concerning the $K_0$ values for applying the initial horizontal stresses is given in Chapter 7. Using the same geometry and material properties of the Selborne experiment and by means of the novel featured developed at Deltares and implemented recently in the code, different initial stresses are generated in the soil. Then the effect of the initial stresses regarding the slip surface generated and the post-failure behaviour in the slope is studied. Also a parametric study of both of the key parameters in the behaviour of the progressive failure modelling is provided.

The concluding remarks are presented in Chapter 8.
2 State of the art

In this chapter a general overview of the existing literature about the recently developed work concerning the slope stability analysis is given and a review of the most important existing research work regarding Finite Elements Methods used to model large deformations is presented. Finally a brief review of the concept of progressive failure, present in brittle materials like over-consolidated clays, is given.

2.1 Slope stability analysis

2.1.1 Introduction

Since the early thirties Heim (1932) and later Terzaghi (1950), landslide researchers had done a lot of effort to predict and understand the slope failures in order to avoid catastrophes. Considerable advances in the understanding of the landslide effects using numerical modelling has been done recently. Although this recent achievements, the extremely rapid motion or the run-out —the resulting post-failure propagation— is nowadays still difficult to predict and object of further research.

Some phenomenological and analytical methods had been studied for predicting the failure character and timing. Moreover, some empirical methods and analytical methods had been developed to model the run-out after the landslide failure. A review of the main existing techniques and quantitative models is given in (Hungr, et al., 2005) and following there are stated these main prediction methodologies.

2.1.2 Prediction methodologies

The fundamental question to answer connected with the landslide risk is what will be the character of the failure: slow and ductile or imminent and in a brittle manner. In case of slow and ductile failure, some urgent actions can be taken in order to avoid the failure, such as stabilization. In case of stabilization is not possible some quickly actions, like evacuation, can be taken. In case of fast deformations the risks are very high and no rapid actions can be taken. The fast deformation can achieve extremely fast velocity of the order of 5 m/s or even faster.

The three possible means to determine the character of the failure include are the following:

- **Judgmental approach:** based on experience and comparison with past episodes. It is known that certain type of soils behave in a brittle or in a ductile manner. Even though, there exist some types of soils than can behave in both manners but with a well-designed typological classification of landslides in preliminary basis permits certain distinction. Further research is developed in (Hungr, et al. 2001)
- **Experimental approach**: based on monitoring the surface displacements which are recorded on time. Then accelerations are analysed in order to predict failures. For practical purposes, a limit of the acceleration or velocity must be set for every unstable slope. For instance, a set of empirical alarms regarding velocities was set in (Salt, 1988) for New Zealand slides in schist. Although further research has been done regarding the experimental approach, they only use the phenomenological base to analyse the failure and overlook the kinematics and causes of the failure.

- **Numerical approach**: based on limit equilibrium or strain-stress analysis. Limit equilibrium techniques examine static stability —they balance the driving forces and the resisting forces within a given slope—. Numerical methods results usually vary depending on the quality of the input data. With a good quality data one can model a potential instability and predict more accurately the failure whereas a limited data can only be used to understand which developing mechanisms may affect the failure.

None of these three approaches are error-proof and specialists usually apply all of them to analyse the failure character.

### 2.1.3 Limit Equilibrium Methods

Limit Equilibrium Analysis (LEA) had become the method of choice when analyzing slope stability as they are simple and a tradition well established method to use in soil mechanics. In the LEA a first assumption of the slip surface has to be made —usually a very geometrical simple surface— and then could be analyzed with different failure criterions for the shear strength of the material. A Mohr-Coulomb yield criterion, prescribing a linear relation between normal and shear stresses is usually taken as the failure criterion in soil mechanics.

Although the LEA does not consider the stress-strain relation of soil, many engineers preferred it, as it provides an estimate of the factor of safety of the slope without knowing the initial conditions. A summary of the review of the existing methods is given in (Mostyn and Small, 1987).

### 2.2 The Finite Element Methods

#### 2.2.1 Introduction

During the last decades, several numerical techniques were used to model solid and fluid deformations. In the Finite Element Methods (FEM) boundary-value problems are approximated by using numerical techniques. At the end of the 19th century Rayleigh (1877) and a little bit later Ritz (1909) already presented how to approximate the boundary-value problem. Nowadays the most used method for finite element formulation is to approximate the solution of the differential equations by means of the weighted residuals method. The weighted residual method includes many approximation techniques such as the sub-domain method (Biezeno, et al., 1933), or the Galerkin method (Galerkin., 1915). Being the last one the most used approximation technique at present.
2.2.2 Lagrangian and Eulerian formulations

In the finite element method formulation exists two basic formulations, named Lagrangian FEM and Eulerian FEM. The Lagrangian FEM is mostly used to model solid deformations whereas the Eulerian FEM is used to model fluid deformations. Following there are described this two formulations highlighting the advantages and drawbacks of each one.

In the Lagrangian FEM the time and material coordinates are used separately to describe the motion or the other physical material properties. In the Eulerian FEM time and spatial coordinates are used together to describe the motion at points fixed in space, while material is passing through with time. The spatial configuration is used as a frame of reference.

In Figure 2.1 it is shown the initial and the deformed configuration for both formulations.

![Figure 2.1](image.png)

**Figure 2.1.** (a) Initial configuration for Eulerian FEM. (b) Deformed configuration for the Eulerian FEM. (c) Initial configuration for Lagrangian FEM and (d) Deformed configuration for Lagrangian FEM

2.2.2.1 Eulerian vs Lagrangian

One of the main advantages of the Lagrangian FEM is that the nodes are always coincident with the material points. Then the nodes located in the boundary always remain in the boundary, therefore the boundary conditions are easily imposed. The other advantage of Lagrangian description is that there is no material allowed to flow between the elements, by definition, and then the elements quadrature remains coincident with material points. Therefore the behaviour can be easily handled. But the main drawback is the mesh distortion when modelling large deformations of the solid.

Eulerian FEM is widely used to model large deformations because the main advantage of this formulation is that there is no mesh distortion. The mesh is kept spatially fixed while the material is deforming. Because of that some drawbacks appear, as the mesh is decoupled from
the material a convective term appears which leads to numerical difficulties due to their non-symmetrical properties. Also some difficulty applying the boundary conditions appears due to the difficulty of model the material interfaces. The convection of the materials it is also a problem because it changes the real physical material properties. Besides, to obtain high quality results in the computations very refined meshes are needed, which increase the computational cost.

2.2.3 Combined Methods

The Eulerian and Lagrangian formulation can be combined; these methods are called Arbitrary Lagrangian Eulerian (ALE) methods (Donea, et al., 2004). The computational mesh can be selected by the user depending on what is wanted to be described. The two formulations can be also used coupled, in geotechnical engineering the Coupled Eulerian-Lagrangian (CEL) methods (Henke, et al., 2010) are used to model penetration problems.

2.2.4 Meshless Methods

Nowadays exists several meshless methods, following there are stated some of them. One of the oldest one is the Smoothed Particle Hydrodynamics (SPH) method (Lucy., 1977). One of the newest is the Element-Free Galerkin (EFG) method (Belytschko, et al., 1994) or the Particle Finite Element Method (PFEM) (Idelsohn, et al., 2004) in which the mesh is done by joining the particles which are represented by the nodes.

The meshless methods are useful for the large deformations problems, but further research is needed regarding the computational efficiency (Belytschko, et al., 1996).

2.3 Progressive Failure

2.3.1 Introduction

Progressive failure is a kind of failure that usually occurs in brittle materials, like over consolidated clays (Skempton, 1964), (Terzaghi, K. & Peck, R.B., 1948), (Bjerrum, 1967) and (Bishop, 1967). In conventional geotechnical engineering situations, argillaceous hard soils and weak rocks fail in a brittle way. For instance, when the peak is reached, strength reduces with increasing deformation and displacement. A very characteristic pattern is usually observed according to (Gens, 2011).

i. first a steep rise in shear stress until reaching the peak strength at low values of displacements is observed,

ii. followed by a rapid reduction of the shear stress, usually associated with the degradation and breakage of the inter-particle bonds designated as post-rupture strength according to (Burland, 1990),

iii. and finally, a more gentle shear stress reduction is observed up to the residual strength, usually associated to a gradual re-alignment of clay particles tending towards the residual sliding shear.

In Figure 2.2 a conceptual scheme for the strength of the argillaceous hard soils and weak rocks is shown, indicating the peak, post-rupture and residual failure envelopes (Gens, 2011). In the first stage of strength loss the bonds of the soil are broken and little or no cohesion will be
governing the failure envelope in the post-rupture stage. In the last stage due to the reorientation of the particles the cohesion and the friction angle are dramatically reduced to the residual strength.

Analytical methods are not really suited to deal with the progressive failure unless one perfectly knows the slip surface beforehand. In the last years the development of new numerical methods as the Finite Element Methods (FEM) had contributed to a better knowledge and understanding of the progressive failure.

Figure 2.2. Conceptual scheme for the strength of the argillaceous hard soils and weak rocks (Gens, 2011).
3 The Material Point Method

In this Chapter the Material Point Method is studied. In the first part how the Dynamic Material Point Method works is discussed according to (Zabala, 2010) and (Jassim, 2013), including the governing differential equations of the phenomenon and how are discretized. Moreover, there is presented a summary of all the steps of the solution procedure of the modified Lagrangian FEM algorithm and how the boundary conditions —zero and non-zero kinematic and traction boundary conditions— are applied within the frame of the Material Point Method.

In the last part, an extension of the Material Point Method to the two-phase problems is presented as the most common in geotechnical engineering is to find problems and situations where coupling between the solid —soil— and fluid —groundwater— phase is present. That coupling means an introduction of considerable complexities to the mechanical behaviour and consequently a high complexity to its numerical simulation. The finite element model for the Velocity formulation ($v$-$w$) and how the two phase dynamic Material Point Method works are presented.

3.1 Dynamic Material Point Method

3.1.1 Introduction

The Material Point Method (MPM) is a numerical technique to model large deformations combining the particle-in-cell methods and Finite Element Methods (FEM), (Sulsky, et al., 1994) & (Sulsky, et al., 1994). In the MPM method the continuum is modelled by Lagrangian points, called material points or particles. The particles carry all the necessary physical variables in order to define the state of the momentum such as mass, material parameters, strains, external loads, etc. Moreover, an Eulerian mesh, which is fixed, is used as a computational mesh to solve the governing equations of the motion by its Gauss points —as in the Lagrangian Finite Elements fashion—. The information is transferred from the particles to the computational mesh at the beginning of every time step. At the end of the step the information is mapped again from the mesh to the particles in order to update the information. Figure 3.1 shows a simplification of how the method works.

Through this approach, MPM combines the advantages of both Lagrangian and Eulerian formulations. It avoids the Eulerian problem of the convective term which generates the numerical diffusion. In addition, it solves the problem of the mesh distortion associated to the Lagrangian mesh working on large deformations.

In (Jassim, 2013) there is presented three novel MPM developments in the analysis of geomechanical problems that involve dynamic problems. Absorbing boundaries are introduced to prevent the reflection of waves at the selected boundary of the domain. The well-known
viscous boundaries, which will continuously creep under load, are modified to viscoelastic boundaries by introducing Kelvin-Voigt elements to limit such non-physical displacements.

![Figure 3.1. How the MPM works](image)

### 3.1.2 Governing differential equations

Considering a given part of the continuum occupies a given volume, \( \Omega_0 \), in an arbitrary initial instant of time, \( t_0 \), and a volume, \( \Omega_t \), in any instant of time \( t \). The volume \( \Omega_0 \) represents the initial state of the continuum and is referred as the initial configuration or the undeformed configuration, whereas the domain \( \Omega_t \) represents the state of the continuum after it has experienced deformation; this domain is referred to as the current configuration or the deformed configuration. In Figure 3.2 the initial and deformed configuration of the continuum are presented.

The material points in the current configuration are denoted by the vector \( \mathbf{X} \) of coordinates and the current position is denoted by the vector \( \mathbf{x} = \varphi(\mathbf{X}, t) \). The mass density for the position \( \mathbf{x} \) in the \( t \) instant of time is denoted by \( \rho(\mathbf{x}, t) \). The vector \( \mathbf{u}(\mathbf{x}, t) = \mathbf{x}(t) - \mathbf{x}(t_0) \) is the displacement and the velocity is denoted by \( \mathbf{v}(\mathbf{x}, t) \); \( \mathbf{\sigma}(\mathbf{x}, t) \) is defined the Cauchy Stress tensor in the position \( \mathbf{x} \) and time \( t \). In Figure 3.3 it is presented the Cauchy Stress tensor in scientific coordinates.

![Figure 3.2. Initial and deformed configuration of a continuum (Oliver, et al., 2000)](image)
3.1.2.1 Conservation of mass

Conservation of mass requires that the material time derivative of the mass be zero for any region of a material volume. Eq. 3.1 presents the local or differential spatial form of mass conservation principle — Continuity Equation — and Eq. 3.2 presents the global spatial form.

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \mathbf{v} = 0 \quad \text{Eq. 3.1}
\]

\[
\frac{d}{dt} \int_{V} \rho \mathbf{v} dV = \int_{V} \left( \frac{d \rho}{dt} + \rho \nabla \cdot \mathbf{v} \right) dV = 0 \quad \text{Eq. 3.2}
\]

3.1.2.2 Linear momentum balance

The time-variation of the linear momentum of a material volume is equal to the resultant force acting on the material volume. If the body is in equilibrium, the linear momentum is conserved. In Eq. 3.3 there is presented the global spatial form for the linear momentum balance whereas in Eq. 3.4 there is presented the so-called Cauchy’s Equation of Motion or the local spatial form for the linear momentum balance, where the \( \mathbf{b} \) vector is the body forces vector.

\[
\int_{V} (\nabla \cdot \mathbf{v} + \rho \mathbf{b}) dV = \int_{V} \frac{d}{dt} \rho \mathbf{v} dV = \int_{V} \rho \frac{d \mathbf{v}}{dt} dV
\]

\[
\nabla \cdot \mathbf{v} + \rho \mathbf{b} = \rho \frac{d \mathbf{v}}{dt} = \rho \mathbf{a}(x,t) \quad \text{Eq. 3.4}
\]

To fulfil the mathematical formulation constitutive laws to relate the stresses of the material with the strains are needed, and is also needed to provide the problem with boundary and initial conditions. Following there are presented the constitutive relation, the boundary conditions and the initial conditions.

3.1.2.3 Constitutive relation

The constitutive relation can be written with respect to the stresses and strains by means of the constitutive tensor \( \mathbf{D} \), Eq. 3.5.

\[
\mathbf{\sigma} = \mathbf{D} : \mathbf{\varepsilon} \quad \text{Eq. 3.5}
\]

In the special case of an isotropic linear elastic material, Hooke’s law is applied, but for non-linear, anisotropic materials the constitutive tensor adopts a difficult and cumbersome expression which generally evolves in time.
3.1.2.4 **Boundary conditions**

The boundary conditions in space affect the spatial arguments of the unknowns and are applied on the contour, \( \partial \Omega = \Gamma \), of the continuum, which is divided into two parts:

i. Prescribed displacements on \( \Gamma_u \)

\[
\mathbf{u}(\mathbf{x}, t) = \mathbf{u}^*(\mathbf{x}, t) \tag{Eq. 3.6}
\]

ii. Prescribed tractions on \( \Gamma_\sigma \)

\[
\mathbf{\sigma}(\mathbf{x}, t) \cdot \mathbf{n} = \mathbf{t}^*(\mathbf{x}, t) \tag{Eq. 3.7}
\]

In Figure 3.4 there are presented the boundary conditions in space. Note there is a third group of boundary conditions where both, displacements and tractions are prescribed on \( \Gamma_{u\sigma} \). Vector \( \mathbf{n} \) is the normal vector to the continuum.

![Figure 3.4. Boundary conditions in space (Oliver, et al., 2000)](image)

3.1.2.5 **Initial conditions**

The initial conditions are the "boundary conditions in time". They affect the time argument of the unknowns and generally they are known values at time zero, \( t_0 \). Eq. 3.8 shows the initial displacements and Eq. 3.9 shows the initial velocity of the momentum.

\[
\mathbf{u}(\mathbf{x}, 0) = 0 \tag{Eq. 3.8}
\]

\[
\left. \frac{\partial \mathbf{u}(\mathbf{x}, t)}{\partial t} \right|_{t=0} = \mathbf{u}(\mathbf{x}, 0) = \mathbf{v}_0(\mathbf{x}) \tag{Eq. 3.9}
\]

3.1.3 **Discretization of the governing equations**

The continuum is modelled by dividing it into elements, each element contains a constant-in-time mass. These elements are represented by a number of materials points or particles, \( N_p \). The mass is assigned to each particle, \( m_p \), with \( p = 1, 2, \ldots, N_p \). The position of each particle in time is noted by \( \mathbf{x}_p(t) \).

The mass density can be written as shows Eq. 3.10, where the mass density is defined as the as a summation of the discrete masses.
The Material Point Method

\[ \rho(x,t) = \sum_{p=1}^{N_p} m_p \delta(x - x_p(t)) \]  

Eq. 3.10

where \( \delta(x - x_p(t)) \) is the Dirac delta function with dimension of inverse of volume.

The shape functions, \( N_i(x) \), associated with spatial nodes, \( x_i(t), i = 1, 2, .., N_n \), where \( N_n \) are the total number of nodes. The shape functions are defined in the nodes of the mesh, and then global position vector, \( x_i \), is then obtained as

\[ x = \sum_{i=1}^{N_n} x_i(t) N_i(x) \]  

Eq. 3.11

The weak form of the Continuity Equation (Eq. 3.1) can be found, based in the standard fashion of FEM —by means of the Galerking weighted residual method (Sulsky, et al., 1994)—, to be

\[ \int_{\Omega} \rho \mathbf{w}_i \cdot \mathbf{a} \, d\Omega = - \int_{\Omega} \rho S \nabla \mathbf{w}_i \cdot d\Omega + \int_{\Gamma_t} \mathbf{w}_i \cdot \mathbf{\tau}d\Gamma + \int_{\Omega} \rho \mathbf{w}_i \cdot \mathbf{b}d\Omega \]  

Eq. 3.12

\[ \mathbf{v} = \sum_{i=1}^{N_n} \mathbf{v}_i(t) N_i(x) \]

\[ \mathbf{a} = \sum_{i=1}^{N_n} \mathbf{a}_i(t) N_i(x) \]

\[ \mathbf{w}_i = N_i \]

By using the expression of the mass density, Eq. 3.10, in the weak form of the Continuity Equation (Eq. 3.12) a discrete equation of the linear momentum conservation is obtained and in that case the integrals are converted into summations of quantities evaluated at the material points.

The integral form of the inertial forces can be written now as

\[ \int_{\Omega} \rho \mathbf{w}_i \cdot \mathbf{a} \, d\Omega = \int_{\Omega} \sum_{p=1}^{N_p} m_p \delta(x - x_p(t)) \cdot N_i(x) \sum_{j=1}^{N_n} \mathbf{a}_j(t) N_j(x) \, d\Omega \]

\[ \int_{\Omega} \rho \mathbf{w}_i \cdot \mathbf{a} \, d\Omega = \sum_{j=1}^{N_n} \sum_{p=1}^{N_p} m_p N_i(x) N_j(x) \mathbf{a}_j(t) \]

Eq. 3.13

and since \( \mathbf{w}_i \) are arbitrary except where the components of displacement are prescribed the weak form of the Continuity Equation (Eq. 3.1) becomes

\[ \sum_{j=1}^{N_n} m^k_j \mathbf{a}_j = f^\text{int,}k + f^\text{ext,}k, \quad i = 1, ..., N_n \]  

Eq. 3.14

where \( m^k_i \) is a lumped mass matrix, \( \mathbf{a}_j \) is the nodal accelerations vector in the node \( j \), and \( f^\text{int} \) and \( f^\text{ext} \) are the internal and external forces vector in the \( i \) node respectively.

The mass matrix is given by

\[ m^k_{ij} = \sum_{p=1}^{N_p} m_p N_i(x^k_p) N_j(x^k_p) \]  

Eq. 3.15
and it varies in time depending on the particles that belongs to the node and it must be computed for every step. In order to reduce the computational cost of the process, a diagonal mass matrix can be used by adding the rows of the consistent mass matrix (Zienkiewicz, et al., 1967).

Using this procedure, the lumped mass matrix becomes

$$m^e_i = \sum_{p=1}^{N_p} m_p N_i(x_p^e)$$  \hspace{1cm} \text{Eq. 3.16}

and equations of motion can be now uncoupled and the following expression can be written for a given node

$$m^e_i a^e_i = f^\text{int.}^e_i + f^\text{ext.}^e_i$$  \hspace{1cm} \text{Eq. 3.17}

or the equivalent expression in terms of momentum conservation

$$\frac{d}{dt} q^e_i = f^\text{ext.}^e_i$$  \hspace{1cm} \text{Eq. 3.18}

where $q^e_i = m^e_i v^e_i$ is the momentum conservation equation for the node $i$ in the $t^e$ instant of time.

### 3.1.4 Solution procedure

First, the particles must be initialized within the background mesh. That means, associate continuum properties —mass, body forces, tractions, etc— to the particles. In (Jassim, 2013) the reader can see the full procedure of the particles initialization of particles information.

In order to solve the equations of motion (Eq. 3.17 & Eq. 3.18) in the Lagrangian frame, these must be initialized and solved at the grid points. One can select implicit or explicit methods for integration of the equations of motion. In FEM, standard Gauss integration is commonly adopted in the space integration. But in the MPM, the continuum is discretized using a finite element number of particles; as a consequence, the integration in space is done by using particles as the integration points instead of Gauss points.

The early MPM solution procedure is identical to the Lagrangian FEM fashion described in (Sulsky, et al., 1994). The identical Lagrangian FEM has some problems when a particle crossing to a previous empty element. Special attention must be paid to this case, illustrated in Figure 3.5, as the value of the shape function will approach to zero. Consequently the node 2 mass will approach zero as well, leading to a nearly ill-condition mass matrix.

The zero mass problem can be solved by introducing a cut-off value to detect small nodal masses. If a nodal mass is slightly smaller than the cut off value, the corresponding internal force is set to be zero. Although this approach seems to be successful, the problem is not solved yet, as the difficulty is now to choose the cut-off criterion.

In order to solve the small mass problem, a slightly modification of this algorithm was proposed later in (Sulsky, et al., 1995) and it has been used in mostly all MPM literature according to Jassim (2013).
3.1.4.1 Overall solution algorithm for a single step (Jassim, 2013)

Herein there is presented a summary of all the steps of the solution procedure of the modified Lagrangian FEM algorithm considering a continuum at time $t$ and advancing the solution to time $t + \Delta t$.

i. The lumped mass matrix $M'$ is computed at the beginning of the time step

$$M' = \sum_{e=1}^{n_{elem}} M_e'. $$

ii. Momentum mapped from the particles to nodes using the shape functions. Then is used to calculate the nodal velocity vector solving the following equation

$$M'v' = \sum_{p=1}^{n_{node}} m_p N^T (\xi_p^t) \dot{v}_p^t. $$

iii. Traction force vector

$$F_{\text{trac},t} = F_{\text{trac},t} \mathbf{T}(t), \quad \text{in which} \quad F_{\text{trac},t} \approx \sum_{p=1}^{n_{node}} N^T (\xi_p^t)\mathbf{f}_{\text{trac}}. $$

iv. Body forces and internal forces are integrated as

$$ F_{\text{grav},t} \approx \sum_{p=1}^{n_{node}} N^T (\xi_p^t) f_{\text{grav}} \quad F_{\text{int},t} \approx \sum_{p=1}^{n_{node}} B^T (\xi_p^t) \sigma_p \Omega_{\sigma_p}. $$

v. Then, the discrete system of equations is complete

$$M'\ddot{v}' = F_{\text{grav},t} + F_{\text{int},t} - F_{\text{trac},t} = F'. $$

vi. The system is now solved for the nodal accelerations

$$\ddot{v}' = M'^{-1}F'. $$

vii. The velocities of particles are now updated using the nodal accelerations and shape functions

$$v_{p}^{t+\Delta t} = v_p^t + \sum_{i=1}^{n_{node}} \Delta t \ N_i (\xi_p^t) \ a_i^t. $$
viii. The nodal velocities are calculated from the updated particle velocities solving the following equation:

\[
M' v_{i}^{t+\Delta t} \approx \sum_{p=1}^{n_{p}} m_{p} N^{T} \left( \xi_{p}^{t} \right) \ddot{u}_{p}^{t+\Delta t}
\]

ix. Nodal velocities are now integrated to get the nodal incremental displacements:

\[
\Delta u_{i}^{t+\Delta t} = \Delta t \ v_{i}^{t+\Delta t}
\]

dx. Strains and stresses at particles are calculated as:

\[
\Delta \varepsilon_{p}^{t+\Delta t} = B \left( \xi_{p}^{t} \right) \Delta u_{i}^{t+\Delta t},
\]

\[
\left\{ \sigma_{p}, \text{material state} \right\}^{t+\Delta t} = \text{constitutive relation} \left\{ \sigma_{p}, \text{material state} \right\}^{t+\Delta t}
\]

xi. Volumes are updated using the volumetric strain increment:

\[
\Omega_{p}^{t+\Delta t} = (1 + \Delta \varepsilon_{vol}^{t+\Delta t}) \ \Omega_{p}^{t}
\]

\[
\Delta \varepsilon_{vol} = \Delta \varepsilon_{11} + \Delta \varepsilon_{22} + \Delta \varepsilon_{33}
\]

xii. Displacements and positions of particles updated according to:

\[
\Delta u_{i}^{t+\Delta t} = \ddot{u}_{i}^{t} + \sum_{i=1}^{n_{e}} N_{i} \left( \xi_{p}^{t} \right) \Delta u_{i}^{t+\Delta t}
\]

\[
x_{p}^{t+\Delta t} = x_{p}^{t} + \sum_{i=1}^{n_{e}} N_{i} \left( \xi_{p}^{t} \right) \Delta u_{i}^{t+\Delta t}
\]

### 3.1.5 Boundary conditions

In the Material Point Method the computational mesh does not align perfectly with the boundary, making the application of the prescribed boundary conditions more cumbersome than in the traditional Lagrangian FEM where the application of them is trivial. Following there is presented the discussion regarding the application of the boundary conditions. The discussion is split into two subsections: zero prescribed boundary conditions and non-zero boundary conditions.

#### 3.1.5.1 Zero kinematic and traction boundary conditions

These boundary conditions are the easiest to apply in the MPM since they are applied in the same way as the Lagrangian FEM fashion. For the zero kinematic boundary conditions a special attention must be paid when applying them, as they must be applied to the nodes that might become active at some point during the computation, too. Regarding zero traction boundary conditions, they are automatically enforced to be satisfied by the solution of equations of motion.

#### 3.1.5.2 Non-zero kinematic and traction boundary conditions

There exists two ways to deal with non-zero boundary conditions. The first one is to map the tractions from the boundary particles to all the nodes where the boundary is located. The clear disadvantage of this procedure is that surface force is distributed through the elements that borders the boundary, so in order to reduce the smearing error, the element size must be very small.

In order to deal with non-zero boundary conditions in a more consistent way, (Jassim, 2013) proposed the concept of the *moving mesh*. The *moving mesh* is a procedure in which the surface tractions and kinematic boundary conditions are applied in a consistent way according to...
Lagrangian FEM so it is considered to be accurate according to the author. Within this procedure the computational mesh is ensured to align with the surface where the tractions are prescribed. In Figure 3.6 there is illustrated the concept of the moving mesh with a block. As one can see, the mesh in front of the block gets compressed while behind of it gets stretched.

For extreme large deformations the distortion of the mesh can be a problem. This problem can be solved by meshing wider zones in the stretched or compressed parts or just re-mesh these zones making use of the inherent MPM feature —the mesh can be modified after each time step—.

![Figure 3.6. Moving mesh procedure](image)

(a) Initial configurations, (b) Deformed configuration with stretched, moving and compressed zones.

(Jassim, 2013)

### 3.2 Dynamic generation and dissipation of pore pressures

#### 3.2.1 Introduction

In geotechnical engineering the most usually is to find problems and situations where the solid —soil— and fluid —groundwater— phases at the same time. That clearly leads to a two-phase problem involving coupling with the solid phase and the fluid phase. The fluid in a porous material introduces considerable complexities to its mechanical behaviour and consequently leads to a high complexity to its numerical simulation.

Herein there is presented the FEM solution procedure for the \( v-w \) formulation according to (Jassim, et al., 2012). Then the differential equations of the hydro-mechanical problem are extended to MPM.

#### 3.2.2 Modelling two-phase problems

The equations describing two-phase problems are well known, and the literature in modelling them on finite element methods is quite extensive. In early studies they focused on the implicit or semi-implicit schemes with high order elements. Nowadays, the studies focused on low order elements with stabilization techniques incorporate circumvent the Babuska-Brezzi or LBB restriction imposed on elements with equal order interpolation for displacement and pore pressure.
In (Esch, et al., 2011) there is presented a detailed comparison regarding the finite element methods that can describe two-phase problems like \( v-p \) formulation (solid velocity – water pressure formulation) and \( v-w \) formulation (solid velocity – water velocity formulation) algorithms. They have shown that although the \( v-p \) formulation can capture dynamic response, that formulation cannot capture dynamic response involving, for instance, the propagation of a compression wave —undrained wave— followed by a second one associated with the consolidation process —damped wave—. On the other hand, they showed that the \( v-w \) formulation can capture the damped wave thanks to that in this formulation all acceleration terms are considered.

The \( v-w \) formulation is capable to precisely capture the response of the soil under dynamic loading. Moreover the \( v-w \) formulation automatically ensures the consistency between pressure and stress. In the next subsection there is presented the finite element model for this formulation.

### 3.2.3 Finite Element Model for \( v-w \) formulation

#### 3.2.3.1 Differential form

Taking tension as positive and given a soil porosity, \( n \), where \( \sigma' \) is the effective stress and \( p \) is the suction pressure. The momentum balance of mixture may be written as

\[
L^T (\sigma' + mp) - (1 - n)\rho_s \dot{v} - np \dot{w} + \rho g = 0
\]

where \( v \) and \( w \) represent the soil particles and fluid velocity respectively. The mixture density

\[
\rho = (1 - n)\rho_s + np_w
\]

is related with the soil density, \( \rho_s \), the water density, \( \rho_w \), and the porosity, \( n \). The momentum balance for the fluid can be written as

\[
\nabla p - \rho_w \dot{w} - \frac{n \gamma_w}{k} (w - v) + \rho_w g = 0
\]

in which \( k \) is the hydraulic conductivity and \( \gamma_w \) is the unit weight of fluid. The velocity of the fluid can be expressed in the more traditional discharge velocity \( q = n (w - v) \). The third term in the Eq. 3.21 corresponds to the interaction between solid and fluid. According to (Esch, et al., 2011) the acceleration of the fluid phase must be retained to capture the dynamic response for general loading.

The mass balance of the mixture is included to fully complete the system of equations as follows

\[
\dot{\rho} = \frac{K_w}{n} [(1 - n) \nabla^T v + n \nabla^T w]
\]

where \( K_w \) is the bulk modulus of the water

### 3.2.4 Two phase dynamic Material Point Method

#### 3.2.4.1 Weak form for FEM and MPM

Using the Galerkin procedure Eq. 3.20 and Eq. 3.21 are converted to the weak form. A linear interpolation for \( v \) and \( w \) is assumed within a finite element such that \( v = Na \) and \( w = Na_w \), where
\( N \) is the corresponding shape function, \( a \) and \( a_n \) are the nodal values of the accelerations for the solid and fluid phase respectively. The momentum balance can be written as

\[
\begin{align*}
M_s \Delta a_s &= \Delta t (R_s - Q(a_s - a))'' \\
M_f \Delta a_f &= \Delta t (R_f - M_s \Delta a_s)
\end{align*}
\]

in which

\[
R_s = F_s - \int_V \mathbf{B}^T \mathbf{m}_p dV \quad \text{and} \quad R_f = F - \int_V \mathbf{B}^T (\mathbf{a}' + \mathbf{m}_p) dV
\]

are the fluid and mixture residual loads, respectively. \( \mathbf{M}_s \) and \( \mathbf{M}_f \) are the corresponding mass matrices for the soil skeleton and water defined respectively as

\[
\mathbf{M}_s = \int_V (1 - n) \rho_s \mathbf{N}^T \mathbf{N} dV \quad \text{and} \quad \mathbf{M}_w = \int_V \rho_w \mathbf{N}^T \mathbf{N} dV.
\]

and

\[
\mathbf{Q} = \int_V \mathbf{n}_w \mathbf{k}^{-1} \mathbf{N}^T \mathbf{N} dV.
\]

### 3.2.4.2 MPM integration

Within the context of FEM, integration is normally carried out using Gauss integrations; for instance

\[
\mathbf{F}_g = \sum_{i=1}^{IP} \mathbf{N}_i^T \rho_i g |\mathbf{J}| w_i \quad \text{and} \quad \mathbf{F}_{int} = \sum_{i=1}^{IP} \mathbf{B}_i^T \mathbf{a}_i |\mathbf{J}| w_i
\]

where \( \mathbf{J} \) is the Jacobian matrix, \( w_i \) are the weighting factor for the integration and IP is the corresponding number of Integration Points.

Within the context of the MPM, information regarding the state of a material volume, \( V_i \), is placed at a particle \( i \). And the MPM equation equivalent to Eq. 3.25 can be written as

\[
\mathbf{F}_g = \sum_{i=1}^{IP} \mathbf{N}_i^T \rho_i g V_i \quad \text{and} \quad \mathbf{F}_{int} = \sum_{i=1}^{IP} \mathbf{B}_i^T \mathbf{a}_i V_i
\]

An important differential issue from the traditional integration approaches is the treatment of fully versus partially filled elements. The stress distribution may lead to spatial oscillations when a particle moving between elements across boundaries. By using Gauss quadrature integration for full elements a smooth stress variation can be obtained. For the elements that are not full regular particle integration is adopted. In Figure 3.7 a comparison between particle integration and Gauss integration is presented for a one-dimensional compression problem, and the smoothing effect of the Gauss integration can be seen clearly.

---

1 An element is considered to be fully filled when the volume sum of all particles inside the element is equal or greater than 90\% of the element volume.
3.2.4.3 Boundary conditions

The boundary of the domain in the $v$-$w$ formulation is the union of the following components

\[
\partial \Omega = \partial \Omega_w \cup \partial \Omega_p = \partial \Omega_u \cup \partial \Omega_p.
\]

where

- $\partial \Omega_w$ is the prescribed fluid velocity boundary
- $\partial \Omega_p$ is the prescribed pressure boundary
- $\partial \Omega_u$ is the soil skeleton prescribed boundary
- $\partial \Omega_r$ is the prescribed total stress boundary

The following conditions should also be satisfied at the boundary

\[
\partial \Omega_u \cap \partial \Omega_r = \emptyset \quad \text{and} \quad \partial \Omega_w \cap \partial \Omega_p = \emptyset
\]

The velocity boundary conditions for the solid and fluid phases and the total traction and pressure boundary conditions can be written as

\[
\begin{align*}
\dot{v}_i(x, t) &= \dot{V}_i(t) & \text{on} \quad \partial \Omega_u(t) \\
\dot{\alpha}_i(x, t) &= \dot{W}_i(t) & \text{on} \quad \partial \Omega_w(t)
\end{align*}
\]

\[
\begin{align*}
\sigma_{ij}(x, t) n_j &= \tau_i(x, t) & \text{on} \quad \partial \Omega_r(t) \\
p(x, t) n_i &= \bar{\rho}_i(x, t) & \text{on} \quad \partial \Omega_p(t).
\end{align*}
\]

Defining

\[
\begin{align*}
\tau_i(x, t) &= \hat{\tau}_i(x) \mathcal{T}(t) & \text{as the prescribed total traction} \\
\bar{\rho}_i(x, t) &= \hat{\rho}(x) n_i \mathcal{T}(t) & \text{as the prescribed pressure}
\end{align*}
\]

And the initial conditions are considered as
\begin{align*}
\dot{\mathbf{v}}(\mathbf{x}, t_0) &= \mathbf{V}_{\text{hs}} \\
\text{and} \\
\dot{\mathbf{w}}(\mathbf{x}, t_0) &= \mathbf{W}_{\text{hs}}, \\
\sigma_{ij}(\mathbf{x}, t_0) &= \sigma_{\text{hs}ij} \\
\text{and} \\
p(\mathbf{x}, t_0) &= p_0.
\end{align*}

Figure 3.8 shows the boundary conditions for a given two-phase problem.

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{boundary_conditions.png}
\caption{Boundary conditions for a given two-phase problem (Jassim, 2013)}
\end{figure}

### 3.2.4.4 Time integration and solution procedure

Herein there is presented the algorithm according to (Verruijt, 2010) algorithm.

1. Eq. 3.23 and Eq. 3.24 are sequentially applied to obtain $\Delta \mathbf{a}_w$ and $\Delta \mathbf{a}$
2. Particle velocities are updated by $\mathbf{v}^{m+1} = \mathbf{v}^m + \mathbf{N} \Delta \mathbf{a}$ and $\mathbf{w}^{m+1} = \mathbf{w}^m + \mathbf{N} \Delta \mathbf{a}_w$
3. Nodal velocities are calculated
   \[ \mathbf{M}_n \mathbf{a}^{m+1}_n = \sum_{i=1}^p \left( (1 - n) \rho_s \mathbf{N} \mathbf{w}^{m+1} \right)_i \text{ and } \mathbf{M}_w \mathbf{a}^{m+1}_w = \sum_{i=1}^p \left( n \rho_w \mathbf{N} \mathbf{w}^{m+1} \right)_i \]
4. Elemental volumetric strain rates are determined for the solid and fluid phases using the already updated nodal velocities
5. Nodal volumetric strain rates are determined and are used to calculate the average strain rates for each phase in each element
6. Pore pressure is updated for each particle $p^{m+1} = p^m + \Delta p$
   \[ \Delta p = \Delta t \frac{K_f}{n} \left[ (1 - n) \left( \mathbf{\bar{e}}_s \right)_s + n \left( \mathbf{\bar{e}}_w \right)_w \right]^{m+1} \]
   Note that although $\Delta p$ is constant in each element, the initial value depends on whether or not the element is fully or partially full, so each value of $p$ for each particle within an element may not be the same. The same applies in for the effective stresses.
7. The effective stresses are updated
   \[ \sigma^{m+1} = \sigma^m + \Delta \sigma' \text{ by recognizing that } \Delta \sigma' = \Delta t \mathbf{D} \mathbf{B} \mathbf{a}^{m+1} \]
8. The displacements of each particle can now be updated by
   \[ \mathbf{u}^{m+1} = \mathbf{u}^m + \Delta t \mathbf{N} \mathbf{a}_w^{m+1} \]

After updating the state parameters, the particles are updated and moved to their new locations and the computational mesh is reset.
3.3 Strain Softening constitutive model

In this section there is presented the basic non-associated Mohr-Coulomb extension presented in Yerro, et al. (2014) by introducing strain softening plasticity with the aim of modelling the strength loss that occurs after peak strength conditions which is used for the analysis of the slope stability analysis.

3.3.1 Mohr-Coulomb Strain Softening law

The Mohr-Coulomb yield surface is written as follows,

\[ q = c' \cos \varphi' + p' \sin \varphi' \quad \text{Eq. 3.28} \]

where

\[ p' = \sigma_1' + \sigma_3'/2 \quad ; \quad p' = \sigma_1' - \sigma_3'/2 \quad \text{Eq. 3.29} \]

\( \sigma_1' \) and \( \sigma_3' \) being the maximum and minimum effective principal stresses respectively.

The softening behaviour is accounted for by allowing the strength parameters (the friction angle, \( \varphi' \), and the cohesion, \( c' \)) to decrease with the accumulated plastic strains, \( \varepsilon_{eq}^p \), according to the softening rules.

\[ c' = c_r' + \left( c_p' - c_r' \right) e^{-\eta \varepsilon_{eq}^p} \quad \text{Eq. 3.30} \]

\[ \varphi' = \varphi_r' + \left( \varphi_p' - \varphi_r' \right) e^{-\eta \varepsilon_{eq}^p} \quad \text{Eq. 3.31} \]

The model requires the specification of peak (\( c_p', \varphi_p' \)) and residual (\( \varphi_r', c_r' \)) strength parameters. Moreover an additional calibration parameter, \( \eta \), is needed. The parameter controls the velocity in which the strength parameters decrease. High values of \( \eta \) lead to a very fast decrease of the strength parameters.

In (Yerro, et al., 2014) the effect of \( \eta \) in a tri-axial test where the vertical strain is prescribed and the confining stress was 10kPa. The results in Figure 3.9 show that high values of \( \eta \) lead to faster degradation of the soil.

![Figure 3.9. Influence of \( \eta \) in the evolution of vertical stresses (Yerro, et al., 2014)](image-url)
3.4 Excavation Feature

A new excavation feature has been implemented in the code, consisting of removing the material points from the geometry given and then it puts all the involved material points into a previously defined element with zero acceleration, velocity, body forces, etc.

In Appendix 1: Excavation Feature, a simple example consisting of removing material considering the geometry given and two different constitutive laws for the material is presented as a kind of tutorial.
4 The Selborne Experiment

In this chapter the Selborne Experiment is summarized based in all the information given in to Cooper (1996), Cooper, et al. (1998) and Grant (1996). Information about geology and geotechnical properties of the site is given. The surcharge system used to increase the pore pressure in order to induce the failure of the slope is presented. Finally, a description of the failure and information about the slip-surface and the displacements achieved during the failure are given.

4.1 Introduction

In this section there is summarized all the information presented in Cooper (1996), Cooper, et al., (1998) and Grant (1996) about the slide experiment in the clay pit of the Selborne Brick and Tile Company's Honey Lane works at Selborne, Hampshire (Figure 4.1). The experiment consisted of a 9 meters deep cut slope in Gault Clay which was induced to failure by a pore pressure increase. The main goal was to study the progressive failure mechanisms generated. For that reason, the site was extensively instrumented by means of piezometers, inclinometers and surface wire extensometer lines.

The slope section used for the study was 25 m wide. First the slope was excavated in the site, in order to induce the two-dimensional displacement low friction panels were installed in at each end of the study section to form isolation trenches. Then by means of wells the pore pressure was increased and the global failure was achieved. It was found that the progressive failure
mechanism of the slope started with movements at the toe in a very early stage of the experiment, even before starting the pore pressure surcharge.

### 4.2 Geology of the site

The site was conveniently chosen because of the geology of the site: a downward succession of soliflucted clay, brown Gault Clay, dark grey Gault Clay and Lower Greensand with a potential brittle behaviour.

In Figure 4.2, all materials found in the slope according to Cooper, et al. (1996) are presented. A thin layer of soliflucted deposits over a layer of weathered Gault Clay. At 7.5 m to 8.5 m depth there is a change from weathered to unweathered dark grey very stiff clay. The base of the Gault Clay lies about 14 m depth. A Lower Greensand was found below 16 meters depth.

### 4.3 Geotechnical properties

At an early stage of the project, continuous undisturbed samples were obtained from two rotary-drilled boreholes located at the crest of the slope. These samples were found to be generally of good quality from which they extracted the geotechnical information of the soil: initial in situ stresses, index properties and shear strength parameters.
$K_0$ values were estimated with a method presented in Skempton (1961), based on capillary pressure measurements. In Figure 4.3 the $K_0$ estimated are presented, which range from 1 to 2. Therefore, the Gault clay was heavily over-consolidated clay.

To determine the effective shear strength parameters drained and un-drained tri-axial tests and shear box tests were carried out. In Table 4.1 the results of these tests are presented.

<table>
<thead>
<tr>
<th>Zone</th>
<th>Peak shear strength</th>
<th>Fully-softened shear strength</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$c'$: kPa</td>
<td>$\phi'$: degrees</td>
</tr>
<tr>
<td>Soliflucted clay</td>
<td>5</td>
<td>21</td>
</tr>
<tr>
<td>Upper weathered clay</td>
<td>10</td>
<td>24</td>
</tr>
<tr>
<td>Lower weathered clay</td>
<td>15</td>
<td>25</td>
</tr>
<tr>
<td>Unweathered Gault Clay</td>
<td>25</td>
<td>26</td>
</tr>
</tbody>
</table>

Table 4.1. Peak and residual shear strength parameters. (Cooper, et al., 1998)

### 4.4 Pore pressure surcharge system

The pore pressure increase was carried out with 20 surcharge wells, arranged in four lines of five wells. Figure 4.4, illustrates the recharge layout sections. Surcharge pressures were first applied in December 1\(^{st}\), 1988 and then raised sequentially to the same level up to 7 meter of water in July 13\(^{th}\) 1989. The failure of the slope occurred in July 16\(^{th}\) of 1989 (196 days after the experiment started). Figure 4.5 shows how the recharge wells progressively modified the overall pore pressure in the slope from Day 2 to Day 184.
4.5 The failure

Thanks to the strong instrumentation of the site, the slip surface could be precisely determined. Two “post-mortem” trenches were excavated in the site; the result of them is shown in Figure 4.6. Note that the assumed plain strain behaviour was not finally reached as the two trenches show different slip surfaces. According to Cooper, et al. (1998) it could be caused by some geological discontinuities. Figure 4.7 shows the interpolated cross-section of the slip surface on the centre line of the slip mass.

According to Cooper, et al. (1998) the evidences shown that the failure and eventual collapse of the slope took place as a result of a progressive failure mechanism. It was initiated rapidly at the toe of the slope and extended into the slope as pore pressures increased. At the same time a similar progressive failure was believed took place starting from the slope crest.

Figure 4.6. Surveyed cross-section of the slip surface
(Cooper, et al., 1998)
The displacement-time curves for the critical elements of all the inclinometers affected by the failure are plotted together in Figure 4.8. A more detailed presentation of the evidence for progressive failure is given in Cooper (1996). It should be noted that at the very first stage of the pore pressure recharge process (Day 2) at least three inclinometers were recording appreciable displacements at the toe of the slope in the eventual slip surface location.

4.6 Recent Modelling of the Experiment

In Castellvi (2015) the Selborne experiment was modelled by using the MPM which contributed to the validation of the method. The post and pre-failure geometry is well known and measured, the parameters of the soil, the pore pressures generated with the surcharge system is well measured with piezometres along the slope. Numerical parameters of the soil were chosen according filed data, and the recharge was modelled and compared with piezometer’s measurements. The progressive failure was simulated and the results showed a final run-out very similar to that observed in the field. However, in that analysis, the excavation process was not simulated and the material was supposed to be normally consolidated and $K_0=0.5$
5 The Selborne Slope Modelling

In the work presented below, a new feature of the MPM code is used to simulate the excavation procedure. With this new tool, the initial stresses can be generated more accurately to the field measurements. The results will be compared with the ones obtained in Castellví (2015). All soil parameters and the geometry used in Castellví (2015) have also been taken as a reference for the sake of comparison.

In this chapter, the pre and post-processing software used for the simulations are introduced. The geometry, boundary conditions, material, loading conditions and the computational mesh are presented. Finally, the different steps performed in the calculation procedure are explained in detail.

5.1 Introduction

The main MPM code used in this work has been developed by the MPM Research Community. The MPM Research Community is a collaboration of currently four partners: Geotechnical and Environmental Research Group of the University of Cambridge, the Soil and Rock Mechanics Research Group of the Universitat Politècnica de Catalunya (UPC), the Institute of Geotechnical Engineering and Construction Management of the Technische Universität Hamburg-Harburg (TUHH) and the Unit Geo-engineering of Deltares.

The MPM formulation used in this work is the one based on Jassim (2013) and it uses an explicit time integration scheme.

5.1.1 Pre-processing software

The software used for pre-processing the geometry, the boundary conditions and the material properties is GiD v11.0.8 developed by CIMNE (International Centre for Numerical Methods in Engineering). GiD is a CAD system that features NURBS (Non-Uniform Rational Basis Spline) surfaces for the geometry definition which provides a set of tools for quick geometry definition. Moreover, GiD allows the generation of meshes in a fast and efficient manner using structured and unstructured meshers for surfaces and volumes.

5.1.2 Post processing software

For the sake of post-processing the results, GiD out v1.0.37 and Praview v4.1.0 is used. ParaView is an open-source data analysis and visualization application. The data exploration can be done interactively in 3D or programmatically using ParaView’s batch processing capabilities.
5.2 Pre-processing

5.2.1 Geometry

The simplified geometry of the Selborne Experiment consists on a slope of 9m high and 63º steep (Figure 5.1). The thickness of the model is considered to be the same as the element used (see the size of the element used in the An excess pore pressure is applied along 24 m of the bottom boundary in order to simulate the water recharge (see Figure 5.2). It is linearly increased during 10 seconds from 0 to 110kPa. Afterwards, the excess pressure on the lower boundary is kept constant.

Mesh section), in order to simulate plane strain behaviour.

5.2.2 Boundary conditions

In order to simulate the plane strain behaviour; the horizontal displacements along the vertical contours are fixed. Moreover, the lower boundary is completely fixed. The water pressure is considered to be zero along the entire ground slope surface and the lateral contours are impermeable. The soil is considered to be totally saturated throughout all the calculation.

5.2.3 Material

The strain softening Mohr-Coulomb constitutive law presented in (Yerro, et al., 2014) is used to model the brittle behaviour of the soil. The material properties assumed while simulating the Selborne experiment are presented in Table 5.1 and Table 5.2.

<table>
<thead>
<tr>
<th>Soil Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Porosity</td>
<td>0.3</td>
</tr>
<tr>
<td>Intrinsic Permeability (m²)</td>
<td>10⁻¹⁰</td>
</tr>
<tr>
<td>Density of the solid (kg/m³)</td>
<td>2700</td>
</tr>
<tr>
<td>Young modulus (kPa)</td>
<td>20000</td>
</tr>
<tr>
<td>Poisson coefficient</td>
<td>0.33</td>
</tr>
<tr>
<td>Calibration parameter for the MC-SS</td>
<td>500</td>
</tr>
</tbody>
</table>

Table 5.1. Soil parameters for the simulation

<table>
<thead>
<tr>
<th>Material</th>
<th>Peak shear Strength</th>
<th>Softened Shear Strength</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$c'$</td>
<td>$\phi'$</td>
</tr>
</tbody>
</table>

Table 5.2. Material properties for the simulation
Table 5.2. Shear strength parameters for the soils

<table>
<thead>
<tr>
<th></th>
<th>[kPa]</th>
<th>[kPa]</th>
<th>[kPa]</th>
<th>[kPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weathered Gault Clay</td>
<td>13</td>
<td>24.5</td>
<td>4.0</td>
<td>13.5</td>
</tr>
<tr>
<td>Gault Clay</td>
<td>25</td>
<td>26</td>
<td>0.5</td>
<td>15</td>
</tr>
</tbody>
</table>

5.3 Loading conditions

An excess pore pressure is applied along 24 m of the bottom boundary in order to simulate the water recharge (see Figure 5.2). It is linearly increased during 10 seconds from 0 to 110kPa. Afterwards, the excess pressure on the lower boundary is kept constant.

5.4 Mesh

The mesh has been refined in the region where the failure is expected in order to get more accurate results and optimise the computational cost. In Figure 5.2 the refined region can be seen. In the area which is not considered the refinement, an unstructured mesh is considered with an element size of 1.5 m; moreover, the surface where the water surcharge is applied is indicated with a blue line. Initially, 4 material points are considered within each element.

In MPM, as well as in finite element methods, the solution of the problem is mesh dependant. This refers to the smallness of the elements required in a model to ensure that the results of an analysis are not affected by changing the size of the mesh. The inclusion of strain-softening features in standard continuum numerical methods leads to mesh dependent strain-localization problems. In this case, the calibration factor $\eta$, plays an important role in order to minimize this dependence (see section Strain Softening constitutive model).

In Figure 5.3 is shown the mesh —with tetrahedral elements— and initial distribution of material points used for the calculations. Note that the mesh is extended more than the slope geometry itself. The MPM needs empty elements where one expects the material points to move. In addition, the same mesh is used for the excavation simulation. In the outer of the refinement zone a bigger element is considered —up to 1.5m— as not really accuracy is needed in that areas.
5.5 Calculation procedure

5.5.1 Particle initialization with the $K_0$ value

The first phase of the calculation is the initialization of the particles with the $K_0$ procedure implemented in the code. The particles can be initialized with a given $K_0$ value and then the excavation of the soil starts.

5.5.2 Excavation

The excavation of the material is done by removing layers of 1.5 m height. All the material points are removed immediately at the beginning of the step and then enough time is left in order to reach the equilibrium as is presented in the following section Reaching the quasi-static equilibrium. The same equilibrium criterion for the out-of-balance forces and the kinetic energy is considered. In Figure 5.4 the slope elevation during the non-dimensional time is presented (See Chapter 6, Introduction for the normalised time definition).

Figure 5.3. Used mesh for the calculations. 1.0m tetrahedral elements with 4 material points.
5.5.3 Reaching the quasi-static equilibrium

The quasi-static equilibrium is reached when both: the out-of-balance forces and the kinetic energy of the whole system vanish. According to Jassim (2013) a dimensionless force ratio, $F$, is defined as:

$$F = \frac{\|F_{\text{ext}} - F_{\text{int}}\|}{\|F_{\text{ext}}\|}$$  \hspace{1cm} \text{Eq. 5.1}

A tolerance of 0.01 is sufficient for both criteria. In Figure 5.5, the dissipation of the out-of-balance forces and kinetic energy error are shown during the first 5 seconds of the calculation. Note that with 5 seconds there is more than enough to reach the quasi-static equilibrium. For the case of non-excavated experiment and in order not generate and accumulate plastic deformations while gravity loading, the gravity has been applied linearly within the 5 initial seconds, from zero to its value.
5.5.4 Increasing the pore pressure in the bottom boundary

In the second phase of the calculation the pore pressure in the base boundary is increased linearly within 10 seconds up to 110kPA. Then, it is kept constant during the rest of the calculation.

5.6 Numerical parameters

There exist different numerical parameters affecting the solution and the computational time: the local damping coefficient and the time step.

5.6.1 Damping

The local damping coefficient is applied to reach convergence to quasi-static equilibrium in a faster way allowing a considerable reduction in the computational time. Figure 5.6, shows the oscillations of the vertical effective stresses while gravity loading for different local damping coefficients. In this case and for the sake of showing how the oscillations occur, the gravity is applied immediately at the beginning of the calculation, in the first step. Note that the final result of the effective vertical stresses for the material point is the same for all cases.

During the excavation phase, a local damping coefficient of 0.75 is considered for that phase because the principal aim is to reach the equilibrium as fast as possible without taking into account dynamic effects. For the rest of the calculation a damping coefficient of 0.05 is considered as in Castellvi (2015).
Figure 5.6. Oscillations of the effective vertical stresses for different damping coefficients while gravity loading

5.6.2 Time step

The numerical methods do not lead to the exact solution of the discretised equations. A numerical method is called stable if these errors stay bounded. It is usual in the dynamic material point method to advance the solution in time using explicit integration schemes. According to Jassim (2013) the algorithm used in this work is a conditionally-stable integration scheme. So the critical time step used here can be written as follows, which is a fraction of the Courant criterion ($\Delta t_{\text{Courant}}$):

$$\Delta t_{\text{crit}} = \alpha_{\text{cour}} \Delta t_{\text{Courant}}$$  \hspace{1cm} \text{Eq. 5.2}

In which $\Delta t_{\text{Courant}} = l/c_p$, $c_p$ is the speed of the compression wave throughout the continuum, $l$ is the minimum element length and $\alpha_{\text{cour}}$ is a reduction factor called $\textit{Courant number}$. 

In the analysis, a Courant number, $\alpha_{\text{cour}}$, of 0.98 is adopted during the excavation phase and while reaching the equilibrium before the water surcharge starts. During the pore pressure increase a smaller time step is needed, as some instabilities of the solution can be seen with a higher time step, so a value of 0.2 is adopted while the pore pressure increases in the bottom boundary.

A time step of $0.22\cdot10^{-2}$ seconds and $0.46\cdot10^{-3}$ seconds are used for the excavation phase and the pore pressure increase phase respectively.
6 Results

In this chapter, the Selborne modelling carried out with the novel excavating feature implemented in the MPM code is presented. These results have been compared with another simulation based on Castellví (2015) in which the initial excavation is not modelled. In addition, they are contrasted with real measured field data. Finally, a complete analysis regarding the progressive failure and the stress paths for several material points along the shear band are presented.

6.1 Introduction

Within this work, a $K_0$ value of 2.0 is taken to generate the horizontal initial stresses and the same analysis—with the same loading conditions, soil parameters, and geometry—is carried out. Then the slip surface and displacements achieved are compared to the simulation carried out in Castellví (2015) and validated against the real results observed in the site and reported in Cooper (1996), Cooper, et al. (1998) and Grant (1996).

For the sake of comparison the time is normalized according to Table 6.1. The normalized time is named $t^*$ whereas the time, $t$, starts counting from the beginning of the experiment. Normalized time from -1 to 0 comprehend the excavation process of the slope. At normalized time 0, the pore pressure surcharge starts. Time 1 is considered to be when all the points along the slip surface have reached the peak strength parameters and the global failure occurs. From 1 to infinity is the time spent by the mobilized soil mass to reach equilibrium and stabilize.

<table>
<thead>
<tr>
<th>Non-Dimensional Interval</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>From -1 to 0</td>
<td>Time spend during excavation and generation of the initial stresses</td>
</tr>
<tr>
<td>From 0 to 1</td>
<td>Time spend in the pore pressure increase until the global failure of the slope is produced</td>
</tr>
<tr>
<td>From 1 to infinity</td>
<td>Time spend after the global failure and the equilibrium is reached</td>
</tr>
</tbody>
</table>

Table 6.1. Non-Dimensional Intervals meaning

6.2 Initial stresses

Herein there are presented the initial stresses generated with the MPM. Figure 6.1 (a) shows the horizontal initial effective stresses whereas (b) shows the vertical initial effective stresses generated on the slope before and after the excavation, on the top before excavation ($t^*=-1.0$) and after excavation ($t^*=0.0$)
On the other hand, Figure 6.2 illustrates the initial stresses with the excavated geometry. The gravity has been applied within 10s so that no oscillations of the dynamic procedure can accumulate plastic deformations of the soil.

It can be seen that the stresses generated without the excavation of the soil (Figure 6.3) are smoother and more uniform than the ones resulting after the excavation procedure (Figure 6.2). That could be due to the disturbances of the unloading conditions generated in the soil while excavating.

### 6.3 Slip surface

In this section, the slip surface generated while the slope is failing is presented. Moreover, a comparison is made between the real observed slip surface and the one obtained without generating the horizontal initial stresses in the soil.

In Figure 6.3, there are two slip surfaces presented. On the left-hand side, the one that is formed with the removal of the soil while on the right, the one generated without the soil excavation (based on Castellví (2015)). In Figure 6.4, a superposition of the estimated slip surface for the both cases is presented and an additional slip surface is superimposed which is the real one observed and measured in the Selborne experiment.
It can be seen in Figure 6.4 that the simulation tackled within this work, the slip surface matches accurately the one observed and measured in the field. It is worth to mention that the initial stresses strongly affect the position of the slip surface in the slope.

Besides, note that the measured slip surface in Selborne is noticeably on top of the simulated slip surfaces. That is because in the Selborne experiment, different ledges were left in order to instrument the slope. The ledge left in the base of the slope added an additional strength to the toe. Those ledges were not simulated in the current experiment so the failure is going to the toe of the slope as is the weakest point.

6.4 Final displacements and velocities

Herein, the horizontal displacements field at the end of the experiment is presented. According to Cooper, et al. (1998), the movement pattern of the surface of the slip mass also included an element of rotation, up to 5°. In Figure 6.5 the results of the simulation and the displacement vectors presented in Grant (1996) are compared. Note that numerical results are very similar to the ones measured in the site.

Moreover, the velocities before the failure were also measured in Grant (1996), just before all the extensometers break due to the large deformations, the velocity measured in the site was 58 mm/h. In the simulations, the maximum speed was reached at 60s ($t^*=1.97$) and was 120mm/s. In Figure 6.7 there are presented the velocities for that instants of time.
6.5 Progressive failure

For a more detailed analysis of the progressive failure, the mobilized shear strength concept is used. The mobilized shear strength concept was introduced in Yerro, et al. (2014) and is a measure of the intensity of shear in a certain point. The concept of mobilized shear friction angle, $\sin \phi'$, is defined in Eq. 6.1.

$$\sin \phi' = \frac{q}{p^*}$$  \hspace{1cm} \text{Eq. 6.1}

Where

$$p^* = p + \frac{c'}{\tan \varphi'}$$
Note that under peak and residual conditions $\phi'$ coincides with the peak and residual friction angles respectively.

In Figure 6.7, there is shown conceptually the progressions of the local shear failure along the developing slip surface where it can be seen how the shear band had been developed with the measures and observations made in Cooper (1996). The concept of the progressions can be studied quantitatively with the mobilized shear strength concept. In Figure 6.7 it can be seen that at a very early stage, the two piezometers located in the toe of the slope already reached peak strength values, even before the start of the pore pressure increase. Then the points located in the crest of the slope start plastifying followed by all the other points located along the shear band and the global failure of the slope is achieved.

![Figure 6.7. Shear strength states around slip surface (Cooper, 1996)](image)

In order to study the failure mechanism in the MPM simulation, 7 material points along the shear band have been taken into account for the analysis (see Figure 6.8).

![Figure 6.8. Points along the shear band object of the study](image)

Figure 6.9 illustrates the evolution in time of the mobilized shear angle for the 7 points along the shear band. Note that time comprehended between -1.0 and 0.0 non-dimensional seconds is the time taken for the excavation before the pore pressure increase starts. It can be seen that the points located in the toe of the slope reach the peak shear strength values before the initiation of the water recharge: first the point located in the toe and then the deeper ones, which are the points P7, P6 and P5, respectively. At $t^* = 0$, just when the water pressure in the slope increases, points P7 and P6 have already reached the residual strength parameters.

In Figure 6.10, the mobilized friction angles during the water recharge are presented. The first material point to reach the peak strength values is P1, located in the crest of the slope, followed rapidly by P2, due to the stress redistribution in that area —at $t^* = 0.42$ and 0.43 respectively—.
After that, P3 and P4 reach the peak strength values at $t^* = 0.58$ and 0.73 respectively. It can be observed that at $t^* = 0.93$ all the points in the shear band are yet in the residual strength values, except for the point P4 that is still decreasing its mobilized friction angle. Finally at $t^* = 1.0$ all the points had reach the residual yielding surface, the global failure of the slope is achieved and the instability is initiated.

Figure 6.9. Mobilized shear angle for the points along the shear band before the experiment starts.

Figure 6.10. Mobilized shear angle along the shear band while the progressive failure is developed.
In Castellví (2015), it was observed that the first points to reach the peak values were the ones located in the crest of the slope. Then the points located in the toe of the slope started reaching the yielding surface and the progressive failure was extended along the shear band. This is the main difference with this current work, where the first points to reach the failure are the ones located in the toe, matching more accurately the real results observed and measured in the site and reported in Cooper (1996), Cooper, et al. (1998) and Grant (1996).

6.6 Stress paths and time evolution of soil parameters

In this section the stress paths and the evolution in time of the different parameters of the soil during the experiment are studied. Two different material points were tracked during the whole experiment: P1 is located in the shear band whereas point P2 is located in the mobilized soil mass. In Figure 6.11, there are shown the location of the two points studied at the beginning of the calculation.

![Figure 6.11. Initial location of the 2 points of study for the excavated geometry](image)

**6.6.1 P1, point located in the shear band**

In Figure 6.12, the time-evolution for the following parameters is studied: excess pore pressures, available cohesion and horizontal displacement of the material point.

In Figure 6.12, it can be seen that when the available cohesion drops to the residual value, the pore pressure value —which is constantly increasing— has a dramatic decrease which is later followed by a lot of oscillations. Those oscillations are attributed to the global failure of the whole slope and are present until the slope reach equilibrium again. Moreover it is also seen that the material point had a small displacement before of the global failure, but it can be noticed that after the global failure that the horizontal displacement increases until a value of 2.0 m.

Furthermore, in Figure 6.13 the stress paths for the material point are shown. For that reason the Lambe \( p' - q \) plane is used. It can be seen that the stress path reach the peak yielding surface and then drops to the residual yielding surface remaining on the residual surface after The mean effective stress and the deviatoric stress are defined as follows respectively.

\[
p' = \frac{\sigma_1 + \sigma_3}{2} \quad \text{Eq. 6.2}
\]

\[
q = \frac{\sigma_1 - \sigma_3}{2} \quad \text{Eq. 6.3}
\]

Moreover, in Figure 6.13, the peak and residual yielding surface are also shown so that it can be seen when the stress path reach the yielding surface and drops to the residual yielding surface. In Figure 6.14 the time evolution of the mean effective stress, \( p' \), and the deviatoric stress, \( q \), is shown.
Figure 6.12. Time evolution of the parameters for material point P1

Figure 6.13. Stress path for material point P1
6.6.2 P2, point located in the mobilized soil mass

Herein the results for the point located in the mobilized soil mass during the global failure are presented. In Figure 6.15 and Figure 6.16 there is shown the time-evolution of the parameters. However, in this case the material point reaches the yielding surface earlier than P1 during the excavation process. Moreover, the oscillations observed in the excess pore pressure are due to the displacements of the soil mass. At the end, all the oscillations are stabilized when the final equilibrium is reached.

In Figure 6.17 there is shown the stress path of P2, it can be seen that when the material point reach the peak yielding surface the strength parameters start decreasing accumulating plastic deformations showing the strength loss that occurs after peak strength conditions in brittle materials. Finally it reaches the residual strength parameters and lies in the residual yielding surface. The oscillations seen in the stresses paths can be observed in time evolution of the $p'-q$ parameters in Figure 6.16.
Figure 6.15. Time evolution of the parameters for material point P2

Figure 6.16. Time evolution of the mean effective stress, $p'$, and the deviatoric stress, $q$, for material point P2
Figure 6.17. Stress path for material point P2
7 Parametric Study

In this chapter, a parametric study concerning the $K_0$ values for applying the initial horizontal stresses is carried out. For this analysis, the same geometry and material properties of the Selborne experiment shown above are used. Different initial horizontal stress field can be generated in the soil. Then the effect of the initial stresses regarding the slip surface generated and the post-failure behaviour in the slope is studied. Moreover, in Castellví (2015) they concluded that a key parameter in the study is the residual cohesion of the material, $c'_{r}$, of the weathered Gault clay. For this reason a further parametric study on the residual cohesion of the weathered clay is provided. Finally, a summary of the intrinsic permeability study is provided according to Castellví (2015).

7.1 Influence of the initial stresses – $K_0$ values

The earth pressure at rest-value ($K_0$) is the ratio of the horizontal effective stress divided by the vertical effective stress (Eq. 7.1). In normally consolidated soils, the effective vertical stress at any depth is assumed to be greater than the horizontal effective stress ($K_0<1$). Meanwhile, in over-consolidated soils, the vertical stresses in the site usually are much greater than the horizontal ones ($K_0>1$).

$$K_0 = \frac{\sigma'_{h0}}{\sigma'_{v0}} \quad \text{Eq. 7.1}$$

In order to study the effect of different degrees of over-consolidation, two different points in the slope are taken into account. In Figure 7.1 there are shown those two different points: P1 is located in an area not affected by the slide motion whereas P2 is located in the area affected by the landslide.

![Figure 7.1. Three different points of study](image)

In the following figures (Figure 7.2 and Figure 7.3) different states have been indicated. State 1 is referred to the initial state after gravity loading and state 2 corresponds to the end of the excavation process which coincides with the beginning of the water recharge. Note that for $K_0=1.0$ an isotropic stress state is applied to the particles, that is why for state 1 (before excavation) the point lies in the isotropic axis of the $p'\cdot q$ plane.

The non-excavated geometry is also shown in Figures 7.2 and 7.3. In this case, both states (1 and 2) coincides as no excavation procedure is simulated. In this case the $K_0$ values are not
applied with the $K_0$ procedure, the geometry is the already excavated geometry and the gravity is applied progressively to generate the initial stresses in the soil. This case is studied because was the procedure that was used in Castellvi (2015) in order to generate the initial stresses.

In Figure 7.2 and Figure 7.3 it can be seen that the excavation process decreases both, the deviatoric and the effective mean stress component in the $p'\cdot q$ plane, except for the case of
$K_0=1.0$ that the deviatoric component has an increase because it was previously (in state 1) lying in the isotropic axis.

Furthermore, it can be seen the effect of the over-consolidation in the soil. As the $K_0$ value increases, the initial stress path starts to be closer to the yielding surface, expecting in this manner a deeper slip surface as the yielding surface will be reached in a faster way when increasing the pore pressure in the lower boundary. Besides, for the non-excavated case Castellví (2015) it can be seen that the starting point also lies close to the yielding surface, so a deeper slip surface is also expected.

### 7.1.1 Slip surfaces comparison

In Figure 7.4 the shear bands observed for each $K_0$ value are presented. Note that the slip surface when $K_0=1.0$ starts at the toe but it does not extend to the rest of the slope. A $K_0$ value of 1.0 locates the particles in the isotropic axis of the $p'-q$ plane, and then a higher deviatoric stress is needed in order to cause the failure. The pore pressure increase applied in this analysis (110 kPa) is not enough to reach the yielding surface in the rest of the points of the slopes, only in the points closer to the bottom. For this reason, the slope remains stable for this configuration.

In Figure 7.5, there are superposed all shear bands observed for all instable cases. It can be observed that whereas $K_0$ is increased the slip surface is deeper in the crest of the slope. The slip surface in the toe is coincident for each of the cases.

### 7.1.2 Kinematic comparison of the failure

In this section the kinematics of the failure are studied. In Figure 7.6 there are presented the final horizontal displacements field for each $K_0$ value. It can be observed that if the instability
occurs, as the $K_0$ value of the soil increases the final run-out of the soil mass decreases. The run-out for the non-excavated case is the largest of each of the cases.

The horizontal displacements in time observed during the failure of the slope are shown in Figure 7.7. It can be seen that as mentioned above, the more over-consolidated is the soil, the less horizontal displacements are observed (note the different scales in the Figure 7.6).

It is important to note that the smaller is the over-consolidated ratio the less time is required to reach the stable geometry (see Figure 7.7).

**Figure 7.6. Horizontal displacements field different $K_0$ values.**

**Figure 7.7. Horizontal displacements in time for a point above the slip surface**
From the different analysis above mentioned, some remarks may be drawn. The initial stresses in the soil, not only affects the slip surface mechanism but also affects in the post-failure behaviour of the landslide. In Figure 7.8 the instant velocities of a material point located above the slip surface during the land sliding are plotted. It can be seen that for smaller values of $K_0$ values —non excavation (0.5) and 1.5— a rapid speed increase is reached and then the velocity slow down and reach equilibrium after some time.

It is observed that in higher over-consolidated soils the post-failure pattern is noticeably different. The process is affected by the stability-reactivation effect shown in Figure 7.7 where subsequent instabilities stages occur. The same effect can be observed in Figure 7.8.

Additionally, the mean velocity for each simulation is computed. A mean velocity of 0.095 m/s, 0.045 m/s and 0.03 m/s is observed for $K_0$ values of 1.5, 2.0 and 2.5 respectively. For the non excavated case a mean velocity of 0.3 m/s is observed.

### 7.2 Residual Cohesion, $c_r'$

In Castellví (2015) it is concluded that one of the key parameters in the modelling is the residual cohesion, $c_r'$, of the weathered Gault clay. As the weathered Gault clay is the material involved in the movement of the soil. In this section, the horizontal displacement values of the mobilised mass, and the slip surface geometries are studied.

In Figure 7.9 there are presented the different horizontal displacements of the soil mass after the failure of the parametric study presented in Castellví (2015). Note that the smaller residual cohesion the longer is the run-out of the material points and the bigger is the area affected by the failure of the slope.
Figure 7.9. Parametric Study for the residual cohesion. (Castellví, 2015),

Figure 7.10. Slip surfaces for the different values of residual cohesion of the weathered clay

Figure 7.11. Run-out results for each residual cohesion
In this work, a parametric study for residual cohesions of the weathered clay is carried out for values varying from 0 to 4.7 kPa. Figure 7.10 illustrates the different slip surfaces generated for the different values of residual cohesion —3, 4, 4.5 and 4.7 kPa—. The real slip surface presented in Cooper, et al. (1998) is superimposed in the same figure —in purple—. For this parametric study the $K_0$ value is taken as 2.0.

It can be seen that the values varying from 4 to 4.7 are the ones that are closer to the real observed surface. Note that the smaller is the residual cohesion value, the failure of the slope happens earlier.
The final horizontal displacements for the different residual cohesion simulation are shown in Figure 7.11. The results for residual cohesion of 4, 4.5 and 4.7 kPa are 3.57, 2.86 and 2.64 m respectively.

The excavation process for a residual cohesion of 0 kPa and 0.5 kPa are illustrated in Figure 7.12. It can be seen how in the 4th and 5th step of the excavation respectively, the slope is already failing. This is due to the stress state present in the material points in that area is really close to the yielding surface. When unloading the soil by the excavation the yielding surface is reached. Moreover the fact that the residual cohesion is really low also affects in this case.

### 7.3 Intrinsic permeability \( \kappa \)

In Castellví (2015), two different simulations were tackled out with two different values, \( 8 \cdot 10^{-11} \) and \( 1 \cdot 10^{-10} \). Castellví concluded that the intrinsic permeability of the soil, \( \kappa \), influences the time when the failure occurs but not the post-failure process, as failure and post-failure stages observed were similar in both cases.

Figure 7.13 shows the time evolution of the stresses, excess pore pressures, and available cohesion for a certain given material point in the slope for both permeability values. Note that the response observed for a lower permeability is the same but earlier in time.

The intrinsic permeability is defined as follows (Eq. 7.2).

\[
\kappa = K \frac{\mu}{\rho g}
\]

\( \mu \) is the dynamic viscosity of the water
\( K \) is the Darcy or hydraulic conductivity
\( \rho \) is the density of the fluid
\( g \) is the gravity

![Figure 7.13. Time evolution of the stresses of a given material point. (Castellví, 2015)](image_url)
8 Conclusions

The material point method (MPM) combines an Eulerian and a Lagrangian description of the dynamic behaviour of materials. In recent years, it has been extended to solve problems in soil mechanics. An MPM code developed to analyse the response of saturated soils (Jassim, et al., 2012) and able to simulate the excavation of soil has been used in this work.

A review of the state of the art regarding the classical and modern methods to study slope stability problems is presented. Moreover, the progressive failure phenomenon, typical in brittle over-consolidated soils, is briefly introduced.

Within this work, the Selborne cutting experiment has been reproduced according to the real results observed in the site presented in Cooper (1996), Cooper, et al. (1998) and Grant (1996). Recently, in Castellví (2015), the same experiment was studied but the initial excavation was not simulated and the material was supposed to be normally consolidated. In this work, the initial stress state has been reproduced much more accurately considering an initial $K_0$ value of 2.0 based on field measurements, and simulating the excavation process of the cut slope.

The results show that the initial failure mechanism obtained here is much more similar to the real one than that presented in Castellví (2015). This fact leads to conclude that the initial stress state highly influences in the geometry of the slip surface. Moreover, the progressive failure mechanism has been correctly reproduced. The first area to reach the peak strength values is the toe of the slope, even before the beginning of the pore pressure increase, due to the excavation effect. Afterwards, when the pore pressure surcharge initiates, the crest area of the slope plastifies and the failure propagates downwards. Finally, the progressive failure is extended to the rest of the slope and the global failure occurs. Those three phases of the progressive failure matched the real results observed in the site.

Additionally, using the same geometry, loading conditions and material parameters, an extended parametric analysis has been carried out in order to study the influence of the initial stresses in the soil. Different $K_0$ values are considered. The results showed that the initial stresses value does not only affect the failure process but also the post-failure behaviour of the mobilized soil mass. The higher the $K_0$, the deeper the initial slip surface. Besides, different pattern for the different $K_0$ values are observed. For lower $K_0$ values a rapid increase of the velocity is observed and then stabilized whereas for higher $K_0$ values the process is affected by the stability-reactivation effect where subsequent instabilities stages occur. For more over-consolidated soils, the final displacements and the velocities achieved are smaller than for the less over-consolidated soils.

This current work contributed to further validate the MPM, and highlighted the potential of the method which is not only capable of describing the conditions leading to slope failure but also, it is capable of following in time the post-failure behaviour of the unstable mass of soil.
Moreover, this work highlighted the importance of the initial stress state modelling in order to approach the correct results in slope stability analysis.
References


The MPM in Slope Stability Analysis


APPENDIXES
A1 Appendix 1: Excavation Feature

Introduction

In this section, the new excavation feature implemented in the MPM code is introduced and a simple example consisting of removing material considering the geometry given and two different constitutive laws for the material is presented as a kind of tutorial.

Geometry

In Figure A I there is presented the geometry for the current example.
Boundary conditions

This is a plane strain simulation, where horizontal displacements are prevented and the bottom boundary has been completely fixed. In Figure A II there are presented the fixities introduced in GiD software to simulate the plain strain conditions.

Figure A II. Boundary conditions

CPS File

In the CPS file, the following new flag names have been introduced:

\[
\text{\$\$APPLY\_EXCAVATION\ ON [ 0 / 1 ]}
\]

If 0 the excavation feature is deactivated, if 1 the code checks if there are material points in the geometry defined in \text{\$\$EXCAVATION\_GEOMETRY} and then it puts all the involved material points in the centre of the element defined in \text{\$\$ELEMENT\_AFTER\_EXCAVATION\ ON} with zero acceleration, velocity, body force, stresses, etc.

\[
\text{\$\$EXCAVATION\_GEOMETRY [ 6 POI NT COORDI NATES ]}
\]

\[
\begin{align*}
X & \ Y & \ Z \ ( P01 \ NT \ 1) \\
X & \ Y & \ Z \ ( P01 \ NT \ 2) \\
X & \ Y & \ Z \ ( P01 \ NT \ 3) \\
X & \ Y & \ Z \ ( P01 \ NT \ 4) \\
X & \ Y & \ Z \ ( P01 \ NT \ 5) \\
X & \ Y & \ Z \ ( P01 \ NT \ 6)
\end{align*}
\]

Here one has to introduce the 6 points that define the volume that is desired to be excavated. It is important to follow the order of the 6 points shown in Figure A III
Appendices

Figure A III. Order of the points of the volume to be excavated

$$\text{ELEMENT\_AFTER\_EXCAVATION} [\text{NUMBER\_OF\_ELEMENT}]$$

XXX

Here the element of the mesh where the material points excavated will be re-allocated when the feature is switched on that should be somewhere in a previously empty element.

**Computational Method**

For the moment the boundary material points are not updated therefore, in this case the computational method has to be set to MPM-MP.

$$\text{COMP\_METHOD}$$

MPM-MP

**Linear Elastic Material Excavation**

In this section the excavation is performed step-wise. First a layer of 1 meter of soil is removed and then 1 meter extra is further excavated, resulting in a column of 3 meters of soil.

**Material**

In the first example a 1 phase linear elastic material it is considered. In Figure A IV there are presented the parameters of the soil used in the calculations.
First step

Apply the gravity load to the whole soil. As the model is a horizontally layered soil the application of the gravity can be done by applying the K0 procedure. On the other hand and for the sake of checking if the new feature is working properly, in the current calculation the gravity is applied with the code.

The local damping procedure with a factor $\alpha = 0.75$ is used in order to get a convergence to equilibrium. In order to check if the equilibrium is reached the force error a threshold of 0.01 is assumed in this tutorial. In Figure A V there is presented how the out-of-balance forces are vanished during the application of the steps. In Figure A VI there are shown the results for the gravitational loading.
Appendixes

Figure A VI. Effective Vertical Stresses after Gravity application [kPa]

Figure A VII. Effective Horizontal Stresses [kPa]
Excavation step

When the first is finished a CPS_002 file is generated. In this current step the upper meter of soil is removed.

$$\text{APPLY EXCAVATION}$$

$$\text{EXCAVATION GEOMETRY}$$

| 1  | 4.0 | 0.1 |
| 1.0 | 5.0 | 0.1 |
| 0.0 | 4.00.0 |
| 1.05.0 | 0.0 |

$$\text{ELEMENT AFTER EXCAVATION}$$

26

The element after excavation can be selected from the GiD software interface. Go to Utilities > List > Elements and select the desired element where the removed material points will be placed. In Figure A VIII there is presented the List dialogue where one can see the element number.

![Figure A VIII. Element list dialogue](image)

The material points belonging to the volume indicated in the CPS File is removed immediately and placed in the element 26. The CPS_002 file should look like as follows.

$$\text{TOTAL TIME}$$

1.00000000000000

$$\text{OVERALL REAL TIME}$$

2.00057419757443

$$\text{COURANT NUMBER}$$
In Figure A IX and Figure A X one can see the resultant stresses after the soil is removed. Note that the material points removed are stored in the centre of the element (in the top right hand of the mesh).

Figure A IX. Vertical Effective Stresses [kPa]
Second step of the excavation

Following the first excavation of soil another layer of one meter is removed following the same procedure above. The .CPS_003 file should look now as follows.

```plaintext
$\$APPLY_EXCAVATION
$\$EXCAVATION_GEOMETRY
  0.00  3.00  0.10
  1.00  3.00  0.10
  1.00  5.00  0.10
  0.00  3.00  0.00
  1.00  5.00  0.00
  0.00  5.00  0.00
$\$ELEMENT_AFTER_EXCAVATION
```

Note that the upper coordinate of Y (5 meters) has been kept to 5 meters in order to excavate the whole soil that is above. Due to the elastic deformation of the soil can rebound upwards and if the volume excavated in this step is only to Y 4 meters some materials point can remain outside this volume and would not be excavated. See the excavated geometry in Figure A XI.

In Figure A XII there are shown the vertical effective stresses of the soil after reaching equilibrium after the second step of the excavation.
Figure A XI. Excavated geometry

Figure A XII. Vertical Effective Stresses [kPa] after 2nd excavation

Cartesian Effective Stress $\sigma_{yy}$
Maximum Value = $7.64 \times 10^3$ kN/m² (Element 26 at Node 825)
Minimum Value = $-57.76$ kN/m² (Element 2701 at Node 6916)
Stress Strain Curve

In Figure A XIV there is presented the Stress-Strain curve. Note that the behaviour is completely linear when loading and unloading.

There is presented the theoretical oedometric modulus:

\[ E' = \frac{1 - \nu}{(1 + \nu)(1 - 2\nu)} E \]
Mohr-Coulomb Material Excavation

In this section the same experiment is carried out with the same geometry, the same boundary conditions but changing the constitutive law of the material for the Mohr Coulomb one. In Figure A XV are shown the material parameters.
Out-of-Balance forces for gravity loading

In Figure A XVI, how the gravity load is applied is shown in time, when applying the gravity in the first step, an oscillation is generated reaching almost 110 kPa, and then the equilibrium is achieved. In Figure A XVI there is presented the \( p'-q \) plane, where it can be seen that the yield criteria in the first step is reached and plastic deformations are accumulated. In the same figure there is also plotted the theoretical \( p'-q \) slope.

\[
M = \frac{6 \sin(\phi)}{3 - \sin(\phi)}
\]

\[
\frac{q}{p'} = \frac{1 - \left(\frac{\nu}{1-\nu}\right)}{\frac{1}{3} \left(1 + \frac{2\nu}{1-\nu}\right)}
\]

In Figure A XVII, there is presented the Stress-Strain curve of the whole excavation. The theoretical oedometric modulus is also plotted.
Figure A XVI. Oscillations of the stresses during the load step

Figure A XVII. Stress-Strain curve
Avoid plastic deformations while gravity loading (M-C Material)

In this section the same experiment has been carried out again changing the Poisson ratio in the material properties in order not to reach the yield criteria when applying gravity. On the other hand the gravity multiplier in the CPS file could be used to apply the gravity progressively from 0 to 1. As way of numerical trick could be increase the peak cohesion during the gravity loading manually in the .GOM file and then change it again before carrying out the second load step calculation.